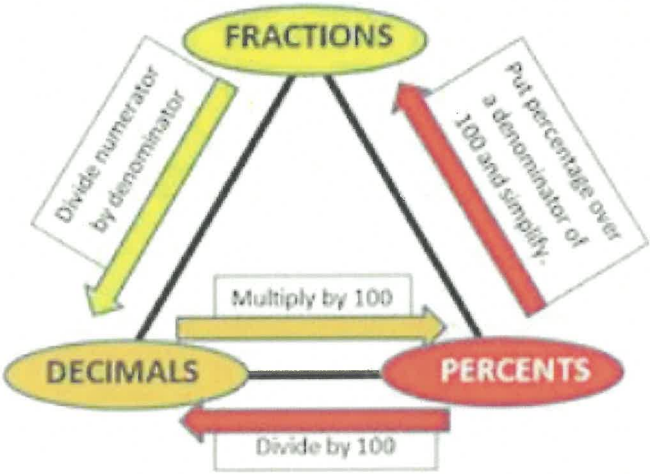
Madani Boys School

Mathematics

YEAR 7

KNOWLEDGE ORGANISER

YEAR 7	
WEEK	**HALF TERM 3**
18	Fractions basics (Equivalent, simplify, convert)
19	F/D/P Conversions / Ordering Fractions, decimals and percentages
20	Fraction Addition and Subtraction
21	Fraction Multiplication and Division
22	Ratio Calculations (simplify, share, recipes)
23	Revision and Assessment 3



FRACTIONS 1

What do I need to be able to do?

You should be able to:

- Understand different representations of fractions
- Fully simplify fractions
- Recognise and find equivalent fractions
- Convert between mixed numbers and improper fractions
- Add/subtract any fractions
- Add/subtract mixed numbers

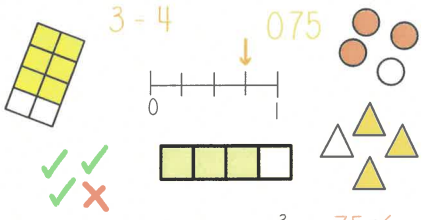
Key Words

- Numerator: the top number of a fraction
- Denominator: the bottom number of a fraction
- Equivalent: of equal value
- Mixed Number: a number with an integer and a proper fraction
- Improper Fraction: a fraction where the numerator is larger than the denominator
- Coprime: two numbers which share no common factors (except 1)

Representing Fractions

numerator $\frac{3}{4}$ denominator

We say 'three quarters' or 'three out of four'



All of these show $\frac{3}{4}$ 75%

Equivalent Fractions

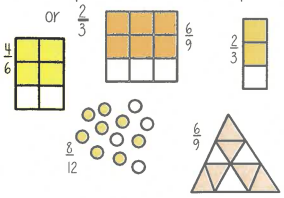
Two fractions are equivalent if they represent the same quantity

If the numerator and denominator have the same multiplier, they are equivalent

$\frac{5}{7} \xrightarrow{\times 5} \frac{25}{35}$ $\frac{1}{4} \xrightarrow{\times 2} \frac{2}{8}$

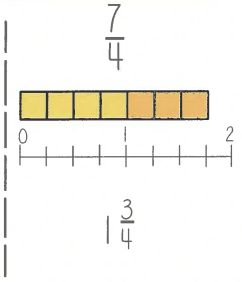
Each of these diagrams represents an equivalent amount

They all show '2 out of every 3' or $\frac{2}{3}$



Mixed Numbers and Improper Fractions

$\frac{3}{2} = 1\frac{1}{2}$ $\frac{10}{3} = 3\frac{1}{3}$



Fractions can represent more than one whole.

The denominator tells us how many parts make up one whole

$\frac{9}{5}$ This tells us that one whole is made up of 5 parts. We have 9 parts, so we can make one whole plus 4 parts

$1\frac{4}{5}$

Simplifying Fractions

You must always simplify you fractions if you can

$\frac{2}{4} \xrightarrow{\div 2} \frac{1}{2}$ HCF of 2 and 4 is 2 so to fully simplify the fraction by dividing the numerator and denominator by 2

$\frac{4}{12} \xrightarrow{\div 2} \frac{2}{6} \xrightarrow{\div 2} \frac{1}{3}$ $\frac{4}{12} \xrightarrow{\div 4} \frac{1}{3}$ Both ways get us to the right answer, just one takes a bit longer!

$\frac{8}{10}$ Sometimes a picture can help to visualise the problem

$\frac{8}{10} = \frac{4}{5}$

Once you cannot find a common factor, the fraction is fully simplified

$\frac{7}{10}$

This fraction is fully simplified as 7 and 10 have no common factors. We can say that 7 and 10 are COPRIME

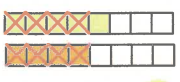
Adding/Subtracting Fractions — Common denominators

$\frac{2}{7} + \frac{4}{7}$

$\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$

Remember that the denominator doesn't change

$\frac{5}{8} - \frac{4}{8}$



$\frac{5}{8} - \frac{4}{8} = \frac{1}{8}$

We can just subtract 4 from 5!

$\frac{1}{10} + \frac{3}{10}$

$\frac{1}{10} + \frac{3}{10} = \frac{4}{10}$

4 and 10 have a common factor (2)!

$= \frac{2}{5}$

You must always fully simplify you fractions

Adding/Subtracting Fractions — Common multiples

$\frac{3}{5} + \frac{1}{10}$

10 is a multiple of 5 (5 x 2) so, using equivalent fractions we can say: $\frac{3}{5} = \frac{6}{10}$

$\frac{6}{10} + \frac{1}{10} = \frac{7}{10}$

$\frac{3}{4} - \frac{1}{12}$

12 is a multiple of 4 (4 x 3) so, using equivalent fractions we can say: $\frac{3}{4} = \frac{9}{12}$

$\frac{9}{12} - \frac{1}{12} = \frac{8}{12} = \frac{2}{3}$

Remember you must always fully simplify your fractions!

$\frac{1}{2} + \frac{2}{3} + \frac{1}{6}$

Here, we know that 2 and 3 share a common multiple of 6, so we can say: $\frac{1}{2} = \frac{3}{6}$ and $\frac{2}{3} = \frac{4}{6}$

$\frac{3}{6} + \frac{4}{6} + \frac{1}{6} = \frac{8}{6}$ Simplify

$= \frac{4}{3}$

We need to give our answer as a mixed number

$= 1\frac{1}{3}$

Adding/Subtracting Fractions — Different denominators

$\frac{1}{5} + \frac{3}{4}$

We need to find a common denominator using equivalent fractions

$\frac{1}{5} = \frac{4}{20}$ $\frac{3}{4} = \frac{15}{20}$

$\frac{19}{20}$

$\frac{3}{11} + \frac{2}{3} = \frac{9}{33} + \frac{22}{33} = \frac{31}{33}$

the LCM of 3 and 11 is 33, so our equivalent fractions are: $\frac{3}{11} = \frac{9}{33}$ $\frac{2}{3} = \frac{22}{33}$

$\frac{5}{7} + \frac{4}{9} = \frac{45}{63} + \frac{28}{63} = \frac{73}{63}$

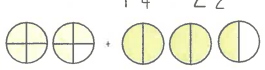
Remember you can find the LCM of 7 and 9 by listing their multiples,
7, 14, 21, 28, 35, 42, 49, 56, 63, 70
9, 18, 27, 36, 45, 54, 63, 72.

Let's convert it to a mixed number

$= 1\frac{10}{63}$

Adding/Subtracting Mixed Numbers

Method 1 $1\frac{3}{4} + 2\frac{1}{2}$



We have three 'wholes' + $\frac{3}{4} + \frac{1}{2}$

$\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4}$

$= \frac{5}{4}$

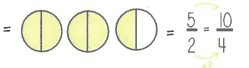
$= 1\frac{1}{4}$

So we have:

$3 + 1\frac{1}{4} = 4\frac{1}{4}$

Method 2 $1\frac{3}{4} + 2\frac{1}{2}$

$1\frac{3}{4} = \frac{7}{4}$ $2\frac{1}{2} = \frac{5}{2} = \frac{10}{4}$



$\frac{7}{4} + \frac{10}{4} = \frac{17}{4} = 4\frac{1}{4}$

How many times does 4 go into 17?
4, 8, 12, 16, 20
4 with a remainder of 1

FRACTIONS 2

What do I need to be able to do?

You should be able to:

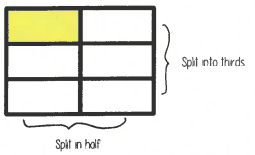
- Multiply unit fractions
- Multiply non-unit fractions
- Use cross-cancelling to simplify fractions before multiplying
- Divide integers by fractions
- Divide fractions by fractions
- Find fractions of amounts
- Use a given fraction to find the whole
- Find the reciprocal of an integer/fraction

Key Words

- Numerator: the top number of a fraction
- Denominator: the bottom number of a fraction
- Unit fraction: a fraction with a numerator of one
- Commutative: changing the order of the operations doesn't change the result
- Reciprocal: the reciprocal of a number is 1 divided by the number
- Coprime: two numbers which share no common factors (except 1)

Multiplying unit fractions

$\frac{1}{2} \times \frac{1}{3}$ "One half of one third"



$= \frac{1}{6}$

Multiplying any fractions

See cross-canceling for a quicker method!

Example 1

$\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$

"Two thirds of two fifths"

Another way to think of it

Two parts out of three parts on two out of five rows

Example 2

$\frac{5}{7} \times \frac{14}{15} = \frac{5 \times 14}{7 \times 15} = \frac{70}{105}$

Remember to simplify where possible! $= \frac{2}{3}$

Example 3 $1\frac{1}{2} \times 2\frac{1}{3}$

$\frac{3}{2} \times \frac{7}{3} = \frac{21}{6} = \frac{7}{2}$

$= 3\frac{1}{2}$

Cross Cancelling Method

$\frac{2}{3} \times \frac{6}{7} = \frac{4}{7}$

This method means that we do not need to simplify our answer as it should be fully simplified already!

Example 1

$\frac{5}{9} \times \frac{18}{25}$

this becomes

$\frac{1}{1} \times \frac{2}{5} = \frac{2}{5}$

Example 2

$\frac{15}{27} \times \frac{36}{45}$

this becomes

$\frac{1 \times 4}{3 \times 3} = \frac{4}{9}$

Remember;
Multiply the numerators then multiply the denominators

Dividing integers by a unit fraction

$3 \div \frac{1}{3}$ Think of this as 'how any times does a third go into 3?'

there are three thirds in one whole, so there are 9 thirds in 3 wholes

Reciprocals

A number multiplied by its reciprocal is always 1

$2 \times \frac{1}{2} = 1$

$5 \times \frac{1}{5} = 1$

The reciprocal of a is $\frac{1}{a}$

Dividing by a fraction, $\frac{1}{a}$, is the same as multiplying by its' reciprocal, a

Example:

$3 \div \frac{1}{3} = 9$

$3 \times 3 = 9$

Dividing Fractions

Example 1

$\frac{2}{3} \div \frac{5}{7}$

$\frac{2}{3} \times \frac{7}{5}$

$\frac{2 \times 7}{3 \times 5}$

$\frac{14}{15}$

Example 2

$\frac{5}{12} \div \frac{25}{18}$

$\frac{5}{12} \times \frac{18}{25}$

$\frac{1 \times 3}{2 \times 5}$

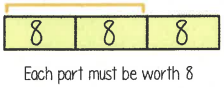
$\frac{3}{10}$

Finding Fractions of Amounts

Find $\frac{1}{2}$ of 10 "Share 10 into 2 equal parts"

Find $\frac{2}{3}$ of 24

$\frac{1}{3}$ is 8 as 24÷3=8



Each part must be worth 8

$\frac{2}{3}$ of 24 = 16

Find $\frac{7}{10}$ of £105

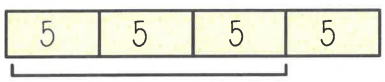
£105 ÷ 10 = £10.50

7 x £10.50 = £73.50

$\frac{7}{10}$ of £105 = £73.50

Reverse Fractions of Amounts

$\frac{3}{4}$ of a number is 15. What is the number?

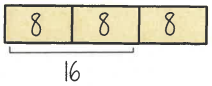


15

If 3 parts = 15, then one part must = 5

The original number was 20

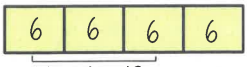
$\frac{2}{3}$ of a number is 16 What is $\frac{3}{4}$ of the number?



16

If 2 parts = 16, then one part must = 8

The number is 8 x3 = 24 So what is $\frac{3}{4}$ of 24?



3 x 6 = 18

One quarter is 6

= 18

Worded problem

A TV is on sale for 2/5 off the price. It now costs £150. How much did it cost originally?

£150

So the original price of the TV was 5 x £50 = £250

FDP EQUIVALENCE

What do I need to be able to do?

You should be able to:

- Convert fluently between fractions, decimals and percentages
- Order fractions, decimals and percentages
- Know the key FDP equivalences

HIGHER TIER ONLY

- Covert recurring decimals into fractions

Key Words

- Percent: parts per hundred
- Fraction: how many parts out of a whole
- Decimal: a number with a decimal point used to separate ones, tenths, hundredths etc...
- Tenth: one whole split into 10 parts
- Equivalent: of equal value
- Recurring decimal: a decimal number with a digit that repeats forever

Percentages to Decimals

Convert 37% to a decimal

Remember this means 37 out of 100 or 37 hundredths. If 1 hundredth is 0.01, 37 hundredths would be 0.37

12% = 0.12 123% = 1.25
85% = 0.85 0.1% = 0.001

Percentage → Decimal, ÷ 100

Percentages to Fractions

Convert 37% to a fraction

Remember this means 37 out of 100. We can write this as $\frac{37}{100}$ ← This is fully simplified!

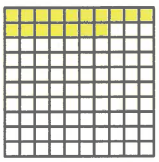
$12\% = \frac{12}{100} = \frac{3}{25}$ $123\% = \frac{123}{100} = 1\frac{23}{100}$

$85\% = \frac{85}{100} = \frac{17}{20}$

Percentage → Fraction, write over 100 and then simplify

Visual aids

Sometimes, it can be helpful to draw a diagram to help understand what is happening



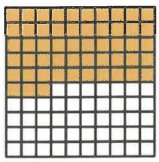
Here are 100 squares I have 17 yellow squares

The fraction of yellow squares is $\frac{17}{100}$

The percentage of yellow squares is 17%

I want to make 0.53 of this big square orange

So I want to make 53 hundredths orange
This is the same as 53% or $\frac{53}{100}$



Decimals to Percentages

Convert 0.63 to a percentage

0.63 is equal to 6 tenths plus 3 hundredths or 63 hundredths. So 0.63 = 63%

0.23 = 23% 0.535 = 53.5%
0.02 = 2% 2.13 = 213%

Decimal → Percentage, x 100

Decimals to Fractions

Always make sure that you fully simplify your fraction!

Convert 0.63 to a fraction

0.63 is equal to 6 tenths plus 3 hundredths or 63 hundredths. We can write this as $\frac{63}{100}$

$0.23 = \frac{23}{100}$ $0.535 = \frac{535}{1000} = \frac{107}{200}$

$0.02 = \frac{2}{100} = \frac{1}{50}$

Here we have 5 tenths, 3 hundredths and 5 thousandths or 535 thousandths

Recurring Decimals to Fractions

HIGHER TIER ONLY

Example (ONE RECURRING DIGIT)

Convert $0.\dot{3}$ to a fraction

$x = 0.3333...$
$10x = 3.3333...$ (10x - x = 9x)
3.333... - 0.333... = 3

$9x = 3 \rightarrow x = \frac{3}{9} = \frac{1}{3}$

Example (TWO RECURRING DIGITS)

Convert $0.\dot{3}\dot{5}$ to a fraction

$x = 0.353535...$
$100x = 35.353535...$

Because we have two digits that are repeating, we need to multiply it by 100!

$99x = 35 \rightarrow x = \frac{35}{99}$

Example

Convert $0.2\dot{5}$ to a fraction

$x = 0.255555...$
$10x = 2.5555555...$
$100x = 25.55555$
25.555 - 2.55 = 23

Here, we cannot just take 2.555 away from 0.255 as we will not reduce it to an integer

100x - 10x = 90x

$90x = 23 \rightarrow x = \frac{23}{90}$

Fractions to Decimals

Convert $\frac{2}{5}$ to a decimal

Remember a divide symbol is an empty fraction, so this is the same as 2 ÷ 5

= 0.4

$$5\overline{)2.0}\quad 0.4$$

Remember

$\frac{1}{10} = 0.1$ $\frac{1}{4} = 0.25$ $\frac{1}{2} = 0.5$

Fractions to Percentages

Convert $\frac{2}{5}$ to a percentage

Here we need to be confident with equivalent fractions. We know percent means out of 100 so we need to find an equivalent fraction with a denominator of 100.

$\frac{2}{5} = \frac{40}{100} = 40\%$

Ordering FDP

Remember ascending means from smallest to largest

Put in ascending order

$0.3, \frac{1}{3}, 0.303, 35\%, \frac{31}{100}$

1. Choose a form to compare them in, here let's choose to compare them as decimals
   0.3, $0.\dot{3}$, 0.303, 0.35, 0.31
2. Put them in ascending order 0.3, 0.303, 0.31, $0.\dot{3}$, 0.35
3. Convert them back to their original form
   0.3, 0.303, $\frac{31}{100}$, $\frac{1}{3}$, 35%

Key FDP Equivalences

www.cazoommaths.com

You are expected to know some of the key FDP equivalences without working them out

Decimal	Percentage	Fraction
0.5	50%	$\frac{1}{2}$
0.25	25%	$\frac{1}{4}$
0.75	75%	$\frac{3}{4}$
0.2	20%	$\frac{1}{5}$
0.1	10%	$\frac{1}{10}$
$0.\dot{3}$	$33.\dot{3}\%$	$\frac{1}{3}$

FDP EQUIVALENCE

What do I need to be able to do?

You should be able to:

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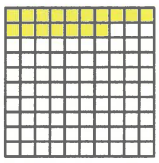

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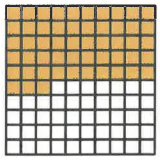

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Decimal → Percentage, x 100

Decimals to Fractions

Always make sure that you fully simplify your fraction!

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Fractions to Decimals

Convert $\frac{2}{5}$ to a decimal

Remember a divide symbol is an empty fraction, so this is the same as 2 ÷ 5

= 0.4

$$\begin{array}{r|l} & 0.4 \\ \hline 5 & 2.0 \end{array}$$

Remember

$\frac{1}{10}$ = 0.1 $\frac{1}{4}$ = 0.25 $\frac{1}{2}$ = 0.5

Fractions to Percentages

Convert $\frac{2}{5}$ to a percentage

Here we need to be confident with equivalent fractions. We know percent means out of 100 so we need to find an equivalent fraction with a denominator of 100

$\frac{2}{5} = \frac{40}{100} =$ 40%

Ordering FDP

Remember ascending means from smallest to largest

Put in ascending order
0.3, $\frac{1}{3}$, 0.303, 35%, $\frac{31}{100}$

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0.3, $0.\dot{3}$, 0.303, 0.35, 0.31

2 Put them in ascending order 0.3, 0.303, 0.31, $0.\dot{3}$, 0.35

3 Convert them back to their original form
0.3, 0.303, $\frac{31}{100}$, $\frac{1}{3}$, 35%

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0.25	25%	$\frac{1}{4}$
0.75	75%	$\frac{3}{4}$
0.2	20%	$\frac{1}{5}$
0.1	10%	$\frac{1}{10}$
$0.\dot{3}$	33.$\dot{3}$%	$\frac{1}{3}$

Example (ONE RECURRING DIGIT)

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$9x = 3 \rightarrow x = \frac{3}{9} = \frac{1}{3}$

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$10x = 2.5555555...$
$100x = 25.55555$

Here, we cannot just take 2.555 away from 0.255 as we will not reduce it to an integer

$25.555 - 2.55 = 23$

$100x - 10x = 90x$

$90x = 23 \rightarrow x = \frac{23}{90}$

YEAR 7 - PROPORTIONAL REASONING...

Ratio and Scale

What do I need to be able to do?

By the end of this unit you should be able to:

- Simplify any given ratio
- Share an amount in a given ratio
- Solve ratio problems given a part

Solutions should be modelled, explained and solved

Keywords

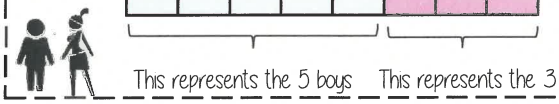
Ratio: a statement of how two numbers compare
Equal Parts:: all parts in the same proportion, or a whole shared equally
Proportion: a statement that links two ratios
Order: to place a number in a determined sequence
Part: a section of a whole
Equivalent: of equal value
Factors: integers that multiply together to get the original value
Scale: the comparison of something drawn to its actual size.

Representing a ratio

"For every 5 boys there are 3 girls"

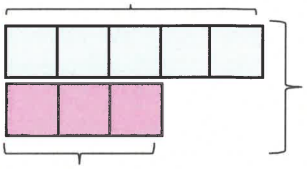
5:3

This is the "whole" – boys and girls together



This represents the 5 boys This represents the 3 girls

Double Number Line



This represents the 5 boys

This is the "whole" – boys and girls together

This represents the 3 girls

Order is Important

"For every dog there are 2 cats"

Dogs: Cats

1:2

The ratio has to be written in the same order as the information is given.
e.g. 2:1 would represent 2 dogs for every 1 cat ✗

Simplifying a ratio

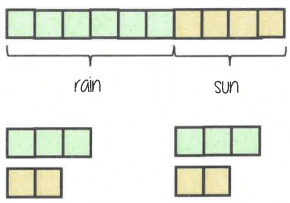
Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"

6:4

÷ by 2

3:2



rain sun

Find the biggest common factor that goes into all parts of the ratio

For 6 and 4 the biggest factor (number that multiplies into them is 2)

"For every 3 days of rain there are 2 days of sun" – when this happens twice the ratio becomes 6:4

Ratio 1:n (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that **this part has to be** 1 unit
Therefore **Divide by 4**

4 : 20
1 : 5

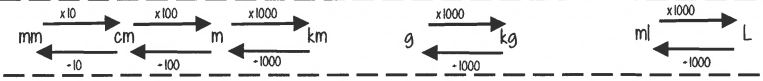
This side has to be divided by 4 too – to keep in **proportion**

H the n part does not have to be an integer for this type of question

Units are important:

When using a ratio – all parts should be in the same units

Useful Conversions



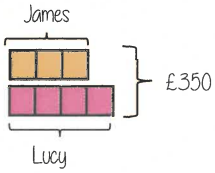
Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4
Work out how much each person earns

Model the Question

James: Lucy

3 : 4



Find the value of one part

Whole £350
7 parts to share between
(3 James, 4 Lucy)

£350 ÷ 7 = £50

□ = one part = £50

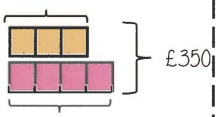
Put back into the question

James: Lucy

3 : 4
(×50 / ×50)
£150 : £200

James = 3 x £50 = £150

Lucy = 4 x £50 = £200



Finding a value given 1:n (or n:1)

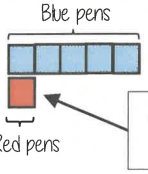
Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red

5 : 1

□ = one part = 10 pens



One unit = 10 pens

Put back into the question

Blue : Red

5 : 1
(×10 / ×10)
50 : 10

Blue pens = 5 x 10 = 50 pens

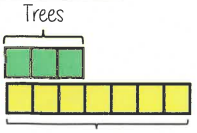
Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

Ratio as a fraction

Trees: Flowers

3 : 7



Ratio

There are 3 parts for trees

Fraction of trees

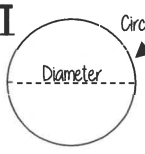
Number of parts of in group / Total number of parts

3/10

Fraction

Tree parts 3 + Flower parts 7 = 10

Pi π



The ratio of a circles circumference to its diameter