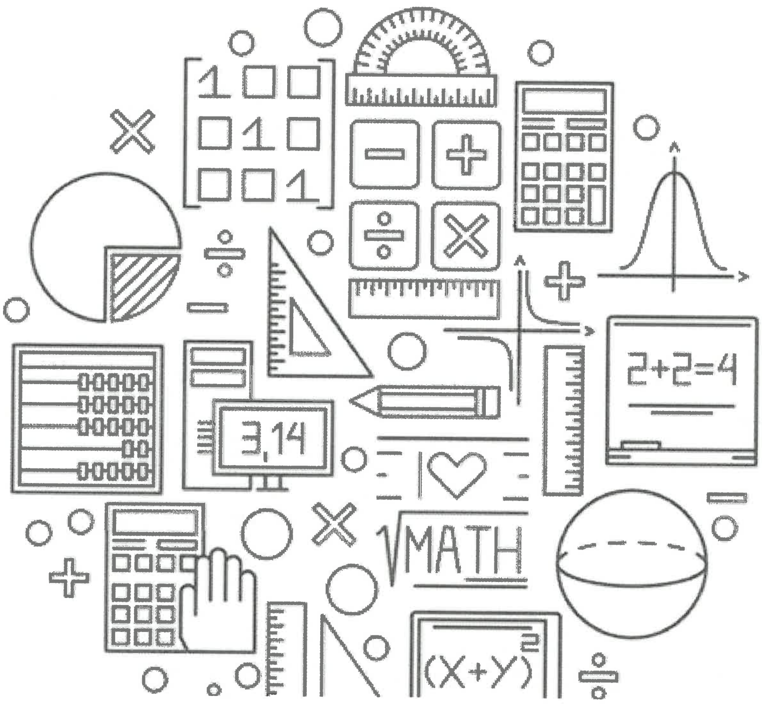
Madani Boys School

Mathematics

YEAR 9

KNOWLEDGE ORGANISER

YEAR 9	
WEEK	**HALF TERM 1**
3	Powers & Roots (negative, fractional)
4	Standard Form
5	Calculations with Standard form
6	
7	Manipulating Surds (simplify, rationalise simple)
8	



INDICES

What do I need to be able to do?

You should be able to:

- Add/subtract with indices
- Multiply expressions with indices
- Divide expressions with indices
- Know the addition law for indices
- Know the subtraction law for indices
- Be familiar with the key results
- Work with negative exponents

HIGHER TIER ONLY

- Work with fractional exponents

Key Words

- Base: the number that gets multiplied by a power
- Power: the number of times the number is used in a multiplication.
- Exponent: power (see above)
- Index: power (see above)
- Coefficient: a number used to multiply a variable
- Variable: a letter which represents an unknown number
- Commutative: changing the order of the operations doesn't change the result

power/exponent/index

coefficient

5^2 | $3d^4$

base

Addition Law for Indices

$$a^m \times a^n = a^{m+n}$$

Examples

- $2^2 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 = \underline{2^5}$
- $k^4 \times k^2 = k \times k \times k \times k \times k \times k = \underline{k^6}$

Subtraction Law for Indices

$$a^m \div a^n = a^{m-n}$$

Examples

$5^3 \div 5 = \frac{5 \times 5 \times 5}{5}$

$= \underline{5^2}$

$d^5 \div d^2 = \frac{d \times d \times d \times d \times d}{d \times d}$

$= \underline{d^3}$

Further Examples

Remember it is best practice to write your variables in alphabetical order!

1. $4w \times 5z = 4 \times 5 \times w \times z = \underline{20wz}$

Multiply the coefficients together and then consider the variables!

2. $3a \times 4a^2 \times 2a$

$= 3 \times 4 \times 2 \times a \times a \times a \times a$

$= \underline{24a^4}$

Remember if there is no power written, it is to the power of 1

3. $(t^3)^2 = t^3 \times t^3$

$= t \times t \times t \times t \times t \times t$

$= \underline{t^6}$

Don't forget that if you square something, you multiply it by itself!

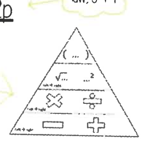
4. $3p^2 \times 4p^3 \div 6p^4 = \frac{3p^2 \times 4p^3}{6p^4}$

$= \frac{12p^5}{6p^4}$

$= \underline{2p}$

Using the subtraction law, 5-4=1

Don't forget about the order of operations!!



Spotting Patterns

$2^3 = 2 \times 2 \times 2 = 8$

$2^2 = 2 \times 2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4} = \frac{1}{2^2}$

$2^{-3} = \frac{1}{8} = \frac{1}{2^3}$

$+1$ $\times 2$ -1 $\div 2$

Each time I add one to the power, I multiply by 2

Each time I take one from the power, I divide by 2

Therefore, 2 to the power of 0 is 1. Remember anything to the power 0 is 1.

If we carry this on, we can even say what 2 to the power of a negative number is!

We can even spot that 2 to the power of -2 is the same as 1 over 2 to the power of 2 (or 2 squared!)

FRACTIONAL INDICES

HIGHER TIER ONLY

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

Examples

$25^{\frac{1}{2}} = \sqrt{25} = \underline{5}$ $8^{\frac{1}{3}} = \sqrt[3]{8} = \underline{2}$

$$a^{\frac{n}{m}} = (\sqrt[m]{a})^n$$

Examples

$25^{\frac{3}{2}} = (\sqrt[2]{25})^3 = 5^3 = \underline{125}$

Remember that this is the same as $(25^{\frac{1}{2}})^3$

Harder Examples

$(81x^2)^{\frac{1}{2}} = \sqrt{81x^2} = \underline{9x}$

$(9c^4)^{\frac{3}{2}} = (\sqrt{9c^4})^3 = (3c^2)^3 = \underline{27c^6}$

Remember this means we cube EVERYTHING inside the brackets

$(32f^{20})^{\frac{3}{5}} = (\sqrt[5]{32f^{20}})^3 = (2f^4)^3 = \underline{8f^{12}}$

It is really helpful to know the powers of 2;

2
4
8
16
32 ..

Refer to the ladder on the right if you're struggling to spot the patterns!

Square and Cube Numbers

When working with indices, it is helpful to know your square and cube numbers!

SQUARE NUMBERS

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225..

CUBE NUMBERS

1, 8, 27, 81, 125, 216, 343, 512...

You are expected to know these!

NEGATIVE FRACTIONAL INDICES

HIGHER TIER ONLY

EXAMPLE 1

$8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}}$

$= \underline{\frac{1}{2}}$

Remember this means the cube root of 8!

EXAMPLE 2

$25^{-\frac{3}{2}} = \frac{1}{25^{\frac{3}{2}}}$

Remember this is the same as $(25^{\frac{1}{2}})^3$

$= \frac{1}{5^3}$

$= \underline{\frac{1}{125}}$

EXAMPLE 3

Don't forget the order of operations!

$(343x^9)^{-\frac{2}{3}} \div x^3$

$= \frac{1}{(343x^9)^{\frac{2}{3}}} \div x^3$

$= \frac{1}{(7x^3)^2} \div x^3$

Remember instead of dividing by x cubed, we can multiply by the reciprocal

$= \frac{1}{49x^6} \times \frac{1}{x^3} = \underline{\frac{1}{49x^9}}$

KEY THINGS TO REMEMBER

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$a^0 = 1$$

$$a^{-m} = \frac{1}{a^m}$$

HIGHER TIER ONLY

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

$$a^{\frac{n}{m}} = (\sqrt[m]{a})^n$$

STANDARD FORM

What do I need to be able to do?

You should be able to :

- Write numbers in standard form
- Convert numbers written in standard form to ordinary numbers
- Order numbers in standard form
- Add/subtract numbers in standard form
- Multiply/divide numbers in standard form
- Use a calculator when working with standard form

Key Words

Standard Form: a system of writing very big or small numbers

Commutative: changing the order of operations doesn't change the result

Base: the number that gets multiplied by a power

Power: the number of times the number is used in a multiplication

Index: power (see above)

Exponent: power (see above)

Negative: a value below zero

Converting ordinary numbers into standard form

$A \times 10^n$

Any integer (n)

Any number between 1 and 10 (A)

Examples

700	12500	0.00034
$= 7 \times 10 \times 10$	must be between 1 and 10 $= 1.25 \times 10 \times 10 \times 10 \times 10$	Remember a negative power doesn't make the answer negative, just closer to 0!
$= 7 \times 10^2$	$= 1.25 \times 10^4$	$= 3.4 \times 10^{-4}$

Converting standard form into ordinary numbers

Example 1

2×10^3

$= 2 \times 10 \times 10 \times 10$

$= 2000$

Example 2

4.12×10^2

$= 4.12 \times 10 \times 10$

$= 412$

Non-Examples

$12 \times 10^2 = 1200$ — must be between 1 and 10!

$1.84 \times 10^{0.9} = 14.62$ — must be an integer!

$6.4 \times 8^3 = 3276.8$ — must be a power of 10!

Index Laws Recap

$10^3 = 10 \times 10 \times 10 = 1000$

$10^2 = 10 \times 10 = 100$

$10^1 = 10$

$10^0 = 1$

$10^{-1} = \frac{1}{10}$

$10^{-2} = \frac{1}{100} = \frac{1}{10^2}$

$10^{-3} = \frac{1}{1000} = \frac{1}{10^3}$

$+1$, $\times 10$

-1, $\div 10$

Each time I add one to the power, I multiply by 10

Each time I take one from the power, I divide by 10

Therefore, 10 to the power of 0 is 1. Remember anything to the power 0 is 1

If we carry this on, we can even say what 10 to the power of a negative number is!

We can even spot that 10 to the power of -2 is the same as 1 over 10 to the power of 2 (or 10 squared!)

Ordering Numbers in Standard Form

10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

3.1×10^2	4.12×10^4	2×10^{-2}	3.281×10^3	2.4×10^{-2}
310	41200	0.02	3281	0.024

$2 \times 10^{-2} \rightarrow 2.4 \times 10^{-2} \rightarrow 3.1 \times 10^2 \rightarrow 3.281 \times 10^3 \rightarrow 4.12 \times 10^4$

Using a Calculator

If we need to write 1.3×10^3 in our calculator;

You're going to need this button here!

Input 1.3 and then press

Then press 3 for the power

Your calculator will often give you the solution to your sum, if it is suitably big/small, in standard form

Adding and Subtracting Numbers in Standard Form

$(2.1 \times 10^6) + (3.3 \times 10^3)$

Foolproof method: convert both numbers to ordinary numbers and then add

$(2.1 \times 10^6) + (3.3 \times 10^3)$

$2,100,000 + 3300$

$= 2,103,300$

$= 2.103 \times 10^6$

You should leave your answer in the form given in the question

$(7.32 \times 10^{-1}) - (2.8 \times 10^{-3})$

$0.732 - 0.0028$

$= 0.7292$

$= 7.292 \times 10^{-1}$

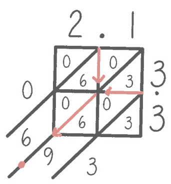
Remember, the best way to work out a subtraction is with column method!

$$\begin{array}{r} 0.7320 \\ -\ 0.0028 \\ \hline 0.7292 \end{array}$$

Multiplying and Dividing Numbers in Standard Form

$(2.1 \times 10^6) \times (3.3 \times 10^3)$

In multiplication and division problems, you can multiply the A values and the look at the powers of 10



$2.1 \times 3.3 \times 10^6 \times 10^3$

$= 6.93 \times 10^6 \times 10^3$

Remember $a^m \times a^n = a^{m+n}$

$= 6.93 \times 10^9$

$(2.8 \times 10^8) \div (7 \times 10^5)$

$\frac{2.8 \times 10^8}{7 \times 10^5} = \frac{0.4 \times 10^8}{10^5} = 0.4 \times 10^3$

Remember $a^m \div a^n = a^{m-n}$

BUT... 0.4×10^3 is not in standard form, as A is not a number between 1 and 10! So...

$0.4 \times 10^3 = 400$

$= 4 \times 10^2$

SURDS

What do I need to be able to do?

You should be able to:

- Simplify surds
- Add and subtract surds, leaving your answer in the simplest form
- Multiply and divide surds, leaving your answer in the simplest form
- Expand brackets involving surds
- Calculate exactly with surds
- Rationalise denominators

Key Words

- Integer: a whole number
- Rational Number: a number which can be expressed in the form $\frac{a}{b}$, where a and b are integers
- Irrational Number: a number which cannot be expressed in the form $\frac{a}{b}$, where a and b are integers
- Expand: multiply out the brackets
- Square Number: the result of multiplying an integer by itself

Examples and Non-Examples

✓ $\sqrt{5}$ $5\sqrt{6}$ $\sqrt{2}$ $3\sqrt{2}$ $\sqrt{3}$ $\sqrt{11}$ $\sqrt{197}$

✗ $\sqrt{4}$ — this can be simplified to 2, which is a rational number

$\sqrt[3]{27}$ — this can be simplified to 3, which is a rational number

$(\sqrt{5})^2$ — this can be simplified to 5, which is a rational number

Adding & Subtracting Surds

$\sqrt{5} + \sqrt{5} = 2\sqrt{5}$ — think of this like x + x, or 2 lots of x

coefficients are dealt with just like they are in algebra!

$4\sqrt{3} + 7\sqrt{3} = 11\sqrt{3}$

$8\sqrt{2} - 5\sqrt{2} = 3\sqrt{2}$

$2\sqrt{3} - 7\sqrt{5}$ — $\sqrt{3}$ and $\sqrt{5}$ are UNLIKE TERMS so this cannot be simplified any further

$4\sqrt{7} + 3\sqrt{10} - \sqrt{7} - 2\sqrt{10} = 3\sqrt{7} + \sqrt{10}$

$\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

$\sqrt{12} = \sqrt{4} \times \sqrt{3}$
$= 2 \times \sqrt{3}$
$= 2\sqrt{3}$

$\sqrt{27} = \sqrt{9} \times \sqrt{3}$
$= 3 \times \sqrt{3}$
$= 3\sqrt{3}$

It is important to try and simplify your surds before working with them so you don't miss things like this!

Multiplying & Dividing Surds

$\sqrt{2} \times \sqrt{5} = \sqrt{2 \times 5} = \sqrt{10}$

$\sqrt{3} \times \sqrt{7} = \sqrt{3 \times 7} = \sqrt{21}$

$\boxed{\sqrt{a} \times \sqrt{b} = \sqrt{ab}}$

$\boxed{\sqrt{a} \times \sqrt{a} = a}$

$\sqrt{2} \times \sqrt{2} = \sqrt{2 \times 2} = \sqrt{4} = 2$

$\sqrt{5} \times \sqrt{5} = \sqrt{5 \times 5} = \sqrt{25} = 5$

Dividing Surds

$\sqrt{10} \div \sqrt{2} = \sqrt{10 \div 2} = \sqrt{5}$

$\boxed{\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}}$

$\sqrt{12} \div \sqrt{3} = \sqrt{12 \div 3} = \sqrt{4} = 2$

Simplifying Surds

Method 1

Simplify $\sqrt{24}$

Here we are looking for the largest square number which is also a factor of 24

Factors of 24
1 x 24
2 x 12
3 x 8
4 x 6

So $\sqrt{24} = \sqrt{4 \times 6}$
$= \sqrt{4} \times \sqrt{6}$
$= 2\sqrt{6}$

Simplify $\sqrt{96}$

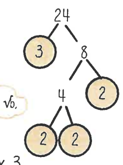
Here we are looking for the largest square number which is also a factor of 96

Factors of 96
1 x 96
2 x 48
3 x 32
4 x 24
6 x 16
8 x 12

So $\sqrt{96} = \sqrt{6 \times 16}$
$= \sqrt{6} \times \sqrt{16}$
$= 4\sqrt{6}$

Method 2

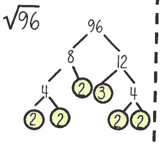
Simplify $\sqrt{24}$



Using prime factor decomposition and our knowledge that $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, we can say

$24 = 2 \times 2 \times 2 \times 3$
So $\sqrt{24} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3}$
$= 2 \times \sqrt{2} \times \sqrt{3}$
$= 2\sqrt{6}$

Simplify $\sqrt{96}$



$96 = 2 \times 2 \times 2 \times 3 \times 2 \times 2$
So $\sqrt{96} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3} \times \sqrt{2} \times \sqrt{2}$
$= 2 \times 2 \times \sqrt{2} \times \sqrt{3}$
$= 4\sqrt{6}$

Expanding brackets

Example 1

Expand and simplify $\sqrt{3}(2 + \sqrt{6})$

X	2	$+\sqrt{5}$
$\sqrt{3}$	$2\sqrt{3}$	$\sqrt{18}$

$\sqrt{18} = \sqrt{9} \times \sqrt{2}$
$= 3\sqrt{2}$

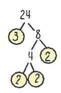
$= 2\sqrt{3} + \sqrt{18}$
$= 2\sqrt{3} + 3\sqrt{2}$

Always remember to check if you can simplify your surds

Example 2

Expand and simplify $\sqrt{3}(3\sqrt{8} - 2\sqrt{2})$

X	$3\sqrt{8}$	$-2\sqrt{2}$
$\sqrt{3}$	$3\sqrt{24}$	$-2\sqrt{6}$



$24 = 2 \times 2 \times 2 \times 3$
So $\sqrt{24} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3}$
$= 2 \times \sqrt{2} \times \sqrt{3}$
$= 2\sqrt{6}$

$= 3\sqrt{24} - 2\sqrt{6}$
$= 6\sqrt{6} - 2\sqrt{6}$
$= 4\sqrt{6}$

Example 3

Expand and simplify $(1 + \sqrt{3})(\sqrt{2} - 1)$

We can treat this just like we do double brackets in algebra!

X	1	$+\sqrt{3}$
$\sqrt{2}$	$\sqrt{2}$	$\sqrt{6}$
-1	-1	$-\sqrt{3}$

none of these are 'like terms' so we cannot simplify anymore!

$= \sqrt{2} - \sqrt{3} + \sqrt{6} - 1$

Rationalising Denominators

Rationalising the denominator means we are making the denominator of the fraction a RATIONAL number (eg, not a surd!).

Example 1

Rationalise the denominator and simplify $\frac{1}{\sqrt{6}}$

We don't want to CHANGE the value of the fraction but we need to find an equivalent fraction with a rational denominator.

We do this by multiplying by '1', in this case,

$\frac{1}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{6}$

Example 2

Rationalise the denominator and simplify $\frac{2}{3 + \sqrt{2}}$

Remember $(x+y)(x-y) = x^2 - y^2$? This result is very important here! We are left with only two square numbers, and we know that means no surds!

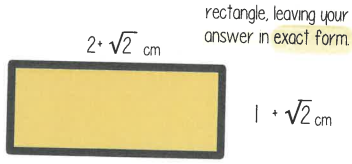
We call (x-y) the conjugate of (x+y), the conjugate of $3 + \sqrt{2}$ is $3 - \sqrt{2}$

$\frac{2}{3 + \sqrt{2}} \times \frac{3 - \sqrt{2}}{3 - \sqrt{2}}$ — Special kind of '1'

$= \frac{2(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})} = \frac{6 - 2\sqrt{2}}{9 - 2} = \frac{6 - 2\sqrt{2}}{7}$

Problem Solving

Calculate the area and perimeter of this rectangle, leaving your answer in exact form.



Perimeter

$1 + \sqrt{2} + 2 + \sqrt{2} + 1 + \sqrt{2} + 2 + \sqrt{2}$

$= 6 + 4\sqrt{2}$ cm

Area

$(1 + \sqrt{2})(2 + \sqrt{2})$
$= 2 + \sqrt{2} + 2\sqrt{2} + 2$
$= 4 + 3\sqrt{2}$ cm²

Exact form means we do not round our answer. This is why surds are so useful as they are an exact value!

Key Things to Remember

$\boxed{\sqrt{a} \times \sqrt{b} = \sqrt{ab}}$ $\boxed{\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}}$

$\boxed{\sqrt{a} \times \sqrt{a} = a}$

$\boxed{\sqrt{a} + \sqrt{a} = 2\sqrt{a}}$ $\boxed{2\sqrt{a} - \sqrt{a} = \sqrt{a}}$

To rationalise the denominator: $\frac{1}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}}$

It is really useful to know your square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225