Year 9 Higher Knowledge Organiser

Half Term 1

Year 9 Higher
CALCULATIONS, CHECKING AND ROUNDING

Key Concepts

A value of 5 to 9 rounds the number up.

A value of 0 to 4 keeps the number the same.

Estimation is a result of rounding to one significant figure.

Examples

Round 3.527 to:

a) 1 decimal place

3 . 5 | 2 7 → 3.5

b) 2 decimal places

3 . 5 2 | 7 → 3.53

c) 1 significant figure

3 | . 5 2 7 → 4

Estimate the answer to the following calculation:

$$\frac{46.2 - 9.85}{\sqrt{16.3 + 5.42}}$$

$$\frac{50 - 10}{\sqrt{20 + 5}}$$

$$\frac{40}{5} = 8$$

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17,56,130

Key Words

Integers
Operation
Negative
Significant figures
Estimate

A) Round the following numbers to the given degree of accuracy

1) 14.1732 (1 d.p.) 2) 0.0568 (2 d.p.) 3)3418 (1 S.F)

B) Estimate:

1) $\sqrt{4.09 \times 8.96}$

2) $25.76 - \sqrt{4.09 \times 8.96}$

3) $\sqrt[3]{26.64} + \sqrt{80.7}$

4) $\frac{\sqrt{6.91 \times 9.23}}{3.95^2 \div 2.02^3}$

ANSWERS A 1) 14.2 2) 0.06 3) 3000 B 1) 6 2) 24 3) 12 4) 4

Year 9 Higher
PERCENTAGES AND INTEREST

Key Concepts

Calculating percentages of an amount without a calculator:

10% = divide the value by 10
1% = divide the value by 100

Per annum is often used in monetary questions meaning **per year.**

Depreciation means that the value of something is going down or reducing.

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93-94

Examples

Simple interest:

Joe invest £400 into a bank account that pays 3% **simple interest** per annum. Calculate how much money will be in the bank account after 4 years.

3% = £4 × 3
= £12
4 years = £12 × 4
Interest = £48
Total in bank account = £400 + £48
= £448

Compound interest:

Joe invest £400 into a bank account that pays 3% **compound interest** per annum. Calculate how much money will be in the bank account after 4 years.

$Value \times (1 \pm percentage\ as\ a\ decimal)^{years}$
$= 400 \times (1 + 0.03)^4$
$= 400 \times (1.03)^4$
= £450.20

Key Words

Percent
Depreciate
Interest
Annum
Simple
Compound
Multiplier

1) Calculate a) 32% of 48 b) 18% of 26
2) Kane invests £350 into a bank account that pays out simple interest of 6%. How much will be in the bank account after 3 years?
3) Jane invests £670 into a bank account that pays out 4% compound interest per annum. How much will be in the bank account after 2 years?

ANSWERS A 1a) 15.36 b) 4.68 2) £413 3) £724.67

Year 9 Higher

COMPOUND INTEREST AND DEPRECIATION

Key Concepts

We use **multipliers** to increase and decrease an amount by a particular percentage.

Percentage increase:

$Value \times (1 + percentage\ as\ a\ decimal)$

Percentage decrease:

$Value \times (1 - percentage\ as\ a\ decimal)$

Appreciation means that the value of something is going up or increasing.

Depreciation means that the value of something is going down or reducing.

Per annum is often used in monetary questions meaning **per year.**

Examples

Compound interest:

Joe invest £400 into a bank account that pays 3% **compound interest** per annum. Calculate how much money will be in the bank account after 4 years.

$Value \times (1 + percentage\ as\ a\ decimal)^{years}$

$= 400 \times (1 + 0.03)^4$

$= 400 \times (1.03)^4$

= £450.20

Compound depreciation:

The original value of a car is £5000. The value of the car **depreciates** at a rate of 7.5% per annum. Calculate the value of the car after 3 years.

$Value \times (1 - percentage\ as\ a\ decimal)^{years}$

$= 5000 \times (1 - 0.075)^3$

$= 5000 \times (0.925)^3$

= £3957.27

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91-92, 94-95

Key Words

Percent
Appreciate
Depreciate
Interest
Annum
Compound
Multiplier

1) Jane invests £670 into a bank account that pays out 4% compound interest per annum. How much will be in the bank account after 2 years?
2) A house has decreased in value by 3% for the past 4 years. If originally it was worth £180,000, how much is it worth now?

ANSWERS A 1) £724.67 2) £159352.71

Year 9 Higher

FRACTIONS, DECIMALS AND PERCENTAGES

Key Concepts

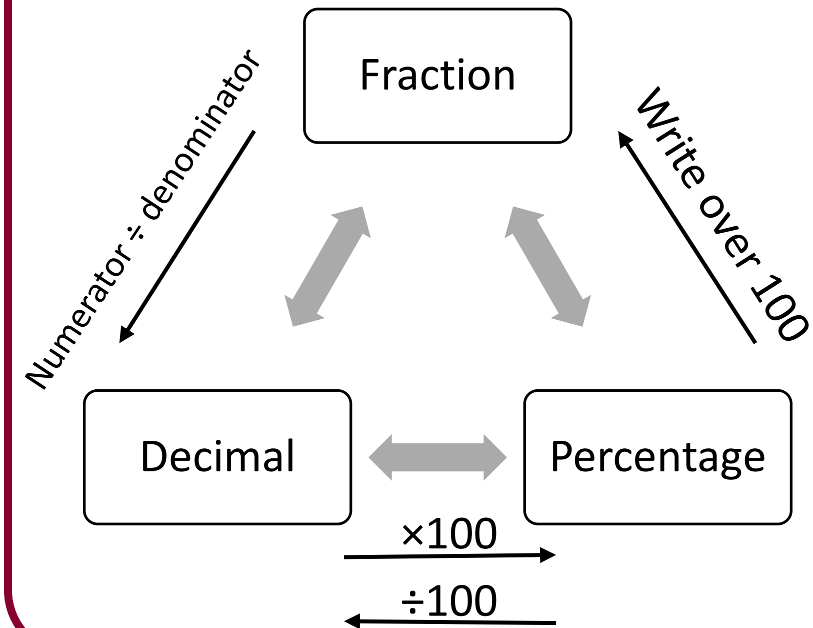
A **fraction** is a numerical quantity that is not a whole number.

A **decimal** is a number written using a system of counting based on the number 10.

Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths
8	7	6	5	.	4	3	2

A **percentage** is an amount out of 100.

Examples



Order the following in ascending order:

$\frac{3}{5}$ 62% 0.67 $\frac{7}{10}$ 0.665

$\frac{3}{5} \xrightarrow{\times 20} \frac{60}{100} \rightarrow 60\%$

$62\% \rightarrow 62\%$

$0.67 \xrightarrow{\times 100} 67\%$

$\frac{7}{10} \xrightarrow{\times 10} \frac{70}{100} \rightarrow 70\%$

$0.665 \xrightarrow{\times 100} 66.5\%$

60% 62% 67% 70% 66.5%

$\frac{3}{5}$ 62% 0.665 0.67 $\frac{7}{10}$

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73-76, 82-83

Key Words

Fraction

Decimal

Percentage

Division

Multiply

1) Convert the following into percentages:

a) 0.4 b) 0.08 c) $\frac{6}{20}$ d) $\frac{3}{25}$

2) Compare and order the following in ascending order:

$\frac{3}{4}$ 76% 0.72 $\frac{4}{5}$ 0.706

ANSWERS 1a) 40% b) 8% c) 30% d) 12% 2) 0.706 0.72 $\frac{3}{4}$ 76% $\frac{4}{5}$

Year 9 Higher
INDICES AND ROOTS

Key Concepts

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}$$

Examples

Simplify each of the following:

1) $a^6 \times a^4 = a^{6+4}$
$= a^{10}$

2) $a^6 \div a^4 = a^{6-4}$
$= a^2$

3) $(a^6)^4 = a^{6\times 4}$
$= a^{24}$

4) $(3a^4)^3 = 3^3 a^{4\times 3}$
$= 27a^{12}$

5) $a^{-3} = \frac{1}{a^3}$

6) $2a^{-4} = \frac{2}{a^4}$

7) $a^{\frac{1}{2}} = \sqrt[2]{a^1} = \sqrt{a}$

8) $a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}} = \frac{1}{\sqrt{a}}$

9) $\left(\frac{25}{16}\right)^{-\frac{1}{2}} = \left(\frac{16}{25}\right)^{\frac{1}{2}}$
$= \sqrt{\frac{16}{25}}$
$= \frac{4}{5}$

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102 – 110

Key Words

Powers
Roots
Indices
Reciprocal

Write as a single power: 1) $a^3 \times a^2$ 2) $b^4 \times b$ 3) $d^{-5} \times d^{-1}$ 4) $m^6 \div m^2$ 5) $n^4 \div n^4$ 6) $\frac{8^4 \times 8^5}{8^6}$ 7) $\frac{4^9 \times 4}{4^3}$

Evaluate : 1) $(3^2)^5$ 2) 2^{-2} 3) $81^{\frac{1}{2}}$ 4) $\left(\frac{1}{9}\right)^{\frac{1}{2}}$ 5) $16^{\frac{3}{2}}$ 6) $27^{-\frac{2}{3}}$

ANSWERS: 1) a^5 2) b^5 3) d^{-6} 4) m^4 5) 1 6) 8^3 7) 4^7
1) 3^{10} 2) $\frac{1}{4}$ 3) 9 4) $\frac{1}{3}$ 5) 64 6) $\frac{1}{9}$

Year 9 Higher
SURDS

Key Concepts

Surds are irrational numbers that cannot be simplified to an integer from a root.

Examples of a surd:
$\sqrt{3}, \sqrt{5}, 2\sqrt{6}$

Examples

Simplify:

$$\begin{aligned} 4\sqrt{20} \times 2\sqrt{3} &= 8\sqrt{20 \times 3} \\ &= 8\sqrt{60} \\ &= 8\sqrt{4}\sqrt{15} \\ &= 16\sqrt{15} \end{aligned}$$

$$\begin{aligned} 3\sqrt{40} \div \sqrt{2} &= 3\sqrt{40 \div 2} \\ &= 3\sqrt{20} \\ &= 3\sqrt{4}\sqrt{5} \\ &= 6\sqrt{5} \end{aligned}$$

Simplify:

$$\begin{aligned} \sqrt{3}(5+\sqrt{6}) &= 5\sqrt{3} + \sqrt{18} \\ &= 5\sqrt{3} + \sqrt{9}\sqrt{2} \\ &= 5\sqrt{3} + 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} (3+\sqrt{2})(4+\sqrt{12}) &= 12 + 4\sqrt{2} + 3\sqrt{12} + \sqrt{24} \\ &= 12 + 4\sqrt{2} + 3\sqrt{4}\sqrt{3} + \sqrt{4}\sqrt{6} \\ &= 12 + 4\sqrt{2} + 6\sqrt{3} + 2\sqrt{6} \end{aligned}$$

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111 – 117

Key Words

Rational

Irrational

Surd

Simplify fully:

1) $2\sqrt{27}$ 2) $2\sqrt{18} \times 3\sqrt{2}$ 3) $\sqrt{72}$ 4) $12\sqrt{56} \div 6\sqrt{8}$

5) $3\sqrt{2}(5 - 2\sqrt{8})$ 6) $(2+\sqrt{5})(3-\sqrt{5})$

ANSWERS: 1) $6\sqrt{3}$ 2) 36 3) $6\sqrt{2}$ 4) $2\sqrt{7}$ 5) $15\sqrt{2} - 24$ 6) $1 + \sqrt{5}$

Year 9 Higher
RATIONALISE THE DENOMINATOR

Key Concepts

A surd can be written within a fraction. However, we do not want an irrational number on the denominator of a fraction therefore we must rationalise it.

To rationalise a surd we can multiply it by itself.

Examples

Rationalise $\frac{1}{\sqrt{5}}$

$$\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{5}}{5}$$

Rationalise $\frac{5}{2\sqrt{3}}$

$$\frac{5}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{5\sqrt{3}}{2 \times 3} = \frac{5\sqrt{3}}{6}$$

Rationalise $\frac{2+\sqrt{3}}{\sqrt{5}}$

$$\frac{2+\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{5}(2+\sqrt{3})}{5}$$

$$= \frac{2\sqrt{5}+\sqrt{15}}{5}$$

Change the sign

Rationalise $\frac{2+\sqrt{3}}{3-\sqrt{5}}$

$$\frac{2+\sqrt{3}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$= \frac{(2+\sqrt{3})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})}$$

$$= \frac{6+3\sqrt{3}+2\sqrt{5}+\sqrt{15}}{9-3\sqrt{5}+3\sqrt{5}-5}$$

$$= \frac{6+3\sqrt{3}+2\sqrt{5}+\sqrt{15}}{4}$$

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118-119

Key Words

Surd

Rationalise

Multiply

Denominator

1) Rationalise $\frac{1}{\sqrt{7}}$

2) Rationalise $\frac{3}{2\sqrt{5}}$

3) Rationalise $\frac{4+\sqrt{5}}{\sqrt{2}}$

4) Rationalise $\frac{2-\sqrt{2}}{1+\sqrt{5}}$

ANSWERS 1) $\frac{\sqrt{7}}{7}$ 2) $\frac{3\sqrt{5}}{10}$ 3) $\frac{4\sqrt{2}+\sqrt{10}}{2}$ 4) $\frac{2+\sqrt{10}-\sqrt{2}-2\sqrt{5}}{-4}$

Half Term 2

Year 9 Higher

EXPRESSIONS/EQUATIONS/IDENTITIES AND SUBSTITUTION

Key Concepts

A **formula** involves two or more letters, where one letter equals an **expression** of other letters.

An **expression** is a sentence in algebra that does NOT have an equals sign.

An **identity** is where one side is the equivalent to the other side.

When **substituting** a number into an expression, replace the letter with the given value.

Examples

1) $5(y + 6) \equiv 5y + 30$ is an identity as when the brackets are expanded we get the answer on the right hand side
2) $5m - 7$ is an expression since there is no equals sign
3) $3x - 6 = 12$ is an equation as it can be solved to give a solution
4) $C = \frac{5(F - 32)}{9}$ is a formula (involves more than one letter and includes an equal sign)
5) Find the value of $3x + 2$ when $x = 5$

   $(3 \times 5) + 2 = 17$
6) Where $A = b^2 + c$, find A when $b = 2$ and $c = 3$

   $A = 2^2 + 3$

   $A = 4 + 3$

   $A = 7$

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153, 154, 189, 287

Key Words

Substitute

Equation

Formula

Identity

Expression

Questions

1) Identify the equation, expression, identity, formula from the list (a) $v = u + at$ (b) $u^2 - 2as$ (c) $4x(x - 2) = x^2 - 8x$ (d) $5b - 2 = 13$

2) Find the value of $5x - 7$ when $x = 3$

3) Where $A = d^2 + e$, find A when $d = 5$ and $e = 2$

ANSWERS: 1) (a) formula (b) expression (c) identity (d) equation 2) 8 3) $A = 27$

Year 9 Higher
EXPANDING AND FACTORISING

Key Concepts

Expanding brackets
Where every term inside each bracket is multiplied by every term all other brackets.

Factorising expressions
Putting an expression back into brackets. To "factorise fully" means take out the HCF.

Difference of two squares
When two brackets are repeated with the exception of a sign change. All numbers in the original expression will be square numbers.

Examples

Expand and simplify:

1) $4(m+5)+3$

$= 4m + 20 + 3$

$= 4m + 23$

2) $(p+2)(2p-1)$

$= p^2 + 4p - p - 2$

$= p^2 + 3p - 2$

3) $(p+3)(p-1)(p+4)$

$= (p^2 + 3p - p - 3)(p+4)$

$= (p^2 + 2p - 3)(p+4)$

$= p^3 + 4p^2 + 2p^2 + 8p - 3p - 12$

$= p^3 + 6p^2 + 5p - 12$

Factorise fully:

1) $16at^2 + 12at \;= 4at(4t+3)$

2) $x^2 - 2x - 3 = (x-3)(x+1)$

3) $6x^2 + 13x + 5$

$= \underbrace{6x^2 + 3x} + \underbrace{10x + 5}$

$= 3x(2x+1) + 5(2x+1)$

$= (3x+5)(2x+1)$

4) $4x^2 - 25$

$= (2x+5)(2x-5)$

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160-166, 168, 169,223-228

Key Words

Expand

Factorise fully

Bracket

Difference of two squares

A)Expand:

1) $5(m-2)+6$ 2) $(5g-4)(2g+1)$ 3) $(y+1)(y-2)(y+3)$

B) Factorise:

1) $5b^2c - 10bc$ 2) $x^2 - 8x + 15$ 3) $3x^2 + 8x + 4$ 4) $9x^2 - 25$

ANSWERS: A 1) $5m - 4$ 2) $10g^2 - 3g - 4$ 3) $y^3 + 2y^2 - 5y - 6$

B 1) $5bc(b-2)$ 2) $(x-3)(x-5)$ 3) $(3x+2)(x+2)$ 4) $(3x+5)(3x-5)$

Year 9 Higher
SOLVING QUADRATICS

Key Concepts

We can solve quadratic equations in 4 different ways:

$$ax^2 + bx + c = 0$$

Factorising – put into brackets first

Completing the square

$$\left(x+\frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2 = 0$$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Graphically

Examples

Factorising:

$$x^2 + 7x + 10 = 0$$
$$(x+2)(x+5) = 0$$
$$Either: x + 2 = 0$$
$$x = -2$$
$$Or: \ x + 5 = 0$$
$$x = -5$$

Completing the square – leave your answer in root form:

$$x^2 + 6x + 5 = 0$$
$$\left(x+\frac{6}{2}\right)^2 + 5 - \left(\frac{6}{2}\right)^2 = 0$$
$$(x+3)^2 + 5 - 3^2 = 0$$
$$(x+3)^2 - 4 = 0$$
$$Either: \ x = \sqrt{4} - 3$$
$$Or: \ x = -\sqrt{4} - 3$$

Quadratic formula – give your answer to 2 decimal places:

$$x^2 + 4x - 2 = 0$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-2)}}{2(1)}$$
$$x = \frac{-4 \pm \sqrt{16+8}}{2}$$
$$Either: \ x = 0.45$$
$$Or: \ x = -4.45$$

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223-228, 235-239, 241-242

Key Words

Solve
Quadratic
Equation
Factorise
Completing the Square
Quadratic formula

1) Solve by factorising: $x^2 + 6x + 8 = 0$
2) Solve by completing the square: $x^2 + 8x + 10 = 0$
3) Solve by using the quadratic formula: $x^2 + 9x - 1 = 0$

ANSWERS: 1) $x = -2, x = -4$ 2) $x = \sqrt{6} - 4, x = -\sqrt{6} - 4$ 3) $x = 0.11, x = -9.11$

Year 9 Higher
REARRANGE AND SOLVE EQUATIONS

Key Concepts

Solving equations:
Working with inverse operations to find the value of a variable.

Rearranging an equation:
Working with inverse operations to isolate a highlighted variable.

In solving and rearranging we **undo the operations** starting from the last one.

Examples

Solve:

$$7p - 5 = 3p + 3$$
-3p -3p
$$4p - 5 = 3$$
+5 +5
$$4p = 8$$
÷ 2 ÷ 2
$$p = 2$$

Solve:

$$5(x - 3) = 4(x + 2)$$
expand expand
$$5x - 15 = 4x + 8$$
$-4x$ $-4x$
$$x - 15 = 8$$
+15 +15
$$x = 23$$

Rearrange to make r the subject of the formulae :

$$Q = \frac{2r - 7}{3}$$
×3 × 3
$$3Q = 2r - 7$$
+7 +7
$$3Q + 7 = 2r$$
÷ 2 ÷ 2
$$\frac{3Q + 7}{2} = r$$

Rearrange to make c the subject of the formulae :

$$2(3a - c) = 5c + 1$$
expand
$$6a - 2c = 5c + 1$$
$+2c$ $+2c$
$$6a = 7c + 1$$
-1 -1
$$6a - 1 = 7c$$
÷ 7 ÷ 7
$$\frac{6a - 1}{7} = c$$

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177-186, 287

Key Words
Solve
Rearrange
Term
Inverse

Links
Science

1) Solve $7(x + 2) = 5(x + 4)$
2) Solve $4(2 - x) = 5(x - 2)$
3) Rearrange to make m the subject $2(2p + m) = 3 - 5m$
4) Rearrange to make x the subject $5(x - 3) = y(4 - 3x)$

ANSWERS: 1) $x = 3$ 2) $x = 2$ 3) $m = \frac{3-4p}{7}$ 4) $x = \frac{4y + 15}{5+3y}$

Year 9 Higher

REARRANGING EQUATIONS

Key Concepts

Rearranging an equation:
Working with inverse operations to isolate a highlighted variable.

In rearranging we **undo the operations** starting from the last one.

Examples

Rearrange to make m the subject of the formulae :

$$m(r+p) = r(h-m)$$

expand   expand

$$mr + mp = rh - mr$$

$+mr$ $+mr$

$$2mr + mp = rh$$

factorise   factorise

$$m(2r+p) = rh$$

$\div (2r+p)$ $\div (2r+p)$

$$m = \frac{rh}{2r+p}$$

Rearrange to make v the subject of the formulae :

$$\frac{1}{f} + \frac{1}{u} = \frac{1}{v}$$

$\times v$ $\times v$

$$\frac{v}{f} + \frac{v}{u} = 1$$

$\times f$ $\times f$

$$v + \frac{fv}{u} = f$$

$\times u$ $\times u$

$$uv + fv = fu$$

factorise   factorise

$$v(u+f) = fu$$

$\div (u+f)$ $\div (u+f)$

$$v = \frac{fu}{u+f}$$

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280-286

Key Words
Rearrange
Term
Inverse
Operation

1) Rearrange to make m the subject $m(c+d) = m + f$

2) Rearrange to make x the subject $\frac{1}{x} = \frac{1}{y} - \frac{1}{z}$

ANSWERS: 1) $m = \frac{f}{c+d-1}$ 2) $x = \frac{yz}{z-y}$

Year 9 Higher
ALGEBRAIC FRACTIONS -SIMPLIFICATION

Key Concepts

To simplify any algebraic fraction we must have a **common term** on the numerator and the denominator.
This will then allow us to **divide through by this term**.

To **multiply** or **divide** algebraic fractions we use the **same principles** as when we calculate with **numerical fractions**.

Examples

Simplify:

$$\frac{x^2+5x}{x^2+7x+10}$$

Factorise the numerator and denominator…

$$\frac{x(x+5)}{(x+2)(x+5)}$$

There should be a repeated term on the numerator and the denominator which can be divided through to leave…

$$\frac{x}{(x+2)}$$

Simplify:

$$\frac{x^2+5x+6}{4} \times \frac{2}{x+2}$$

$$\frac{2(x^2+5x+6)}{4(x+2)}$$

Factorise…

$$\frac{2(x+3)(x+2)}{4(x+2)}$$

Divide through by $(x+2)$ to leave…

$$\frac{2x+6}{4} = \frac{x+3}{2}$$

Simplify:

$$\frac{4}{x-2} \div \frac{3}{x^2-2x}$$

Do the reciprocal of the 2nd fraction and multiply…

$$\frac{4}{x-2} \times \frac{x^2-2x}{3} = \frac{4(x^2-2x)}{3(x-2)}$$

Factorise…

$$\frac{4x(x-2)}{3(x-2)}$$

Divide through by $(x-2)$ to leave…

$$\frac{4x}{3}$$

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170,229

Key Words

Simplify
Numerator
Denominator
Factorise
Divide
Multiply

Simplify:

1) $\frac{x^2+6x+9}{x^2-2x-15}$ 2) $\frac{4}{x-2} \times \frac{x^2-2x}{8}$ 3) $\frac{x^2+7x+10}{2} \div \frac{x^2+4x-5}{4}$

ANSWERS 1) $\frac{x+3}{x-5}$ 2) $\frac{x}{2}$ 3) $\frac{2x+4}{x-1}$

Year 9 Higher
ALGEBRAIC FRACTIONS -SOLVING

Key Concepts

An algebraic fraction can be set equal to a value. When this occurs we are able to **solve the equation** and find out the **value of the unknown term**.

If two algebraic fractions are involved we combine them to make one using the rules of the four operations of fractions.

Examples

Solve:

$$\frac{x}{x-3}+\frac{4}{x+2}=2$$

Add the fractions by finding a common denominator…

$$\frac{x(x+2)+4(x-3)}{(x-3)(x+2)}=2$$

Expand your brackets and simplify…

$$\frac{x^2+2x+4x-12}{x^2-3x+2x-6}=2$$

$$\frac{x^2+6x-12}{x^2-x-6}=2$$

Multiply both sides by the denominator…

$$x^2+6x-12=2x^2-2x-12$$

Rearrange to have the equation equal zero…

$$x^2-8x=0$$

Solve the quadratic by either factorising, using the quadratic formula or completing the square…

$$x(x-8)=0$$

Either:

$$x=0$$

Or:

$$x-8=0$$
$$x=8$$

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187, 244

Key Words

Solve
Expand
Factorise
Rearrange
Quadratic
Formula

1) Solve using factorising:

$$\frac{3}{x+2}+\frac{2}{x+4}=2$$

2) Solve using the quadratic formula:

$$\frac{1}{2x-1}+\frac{2}{x+5}=1$$

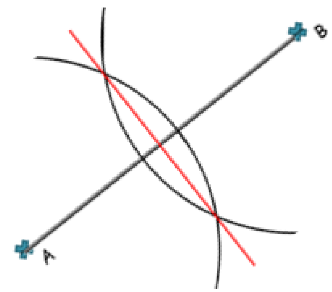
ANSWERS 1) $x=0$ *or* $x=-3.5$ 2) $x=-3.24$ *or* $x=1.24$

Half Term 3

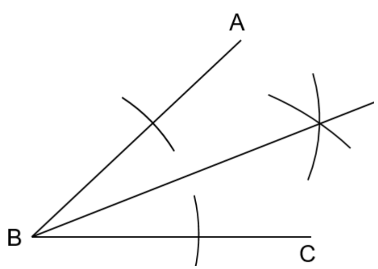
Year 9 Higher
CONSTRUCTIONS AND LOCI

Key Concepts

Line bisector



Angle bisector



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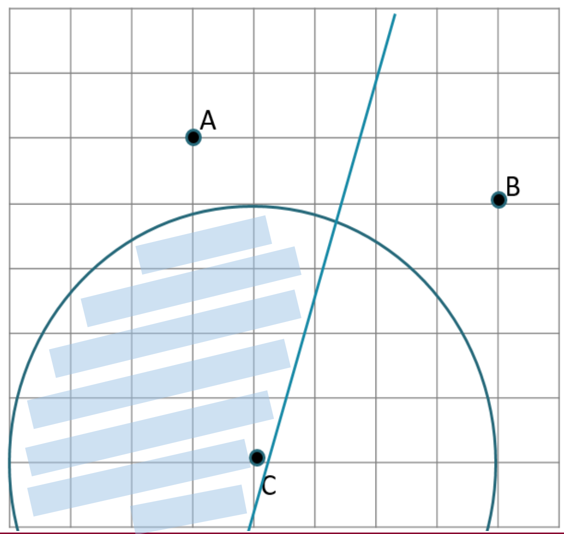
683,660-665, 674-679

Examples



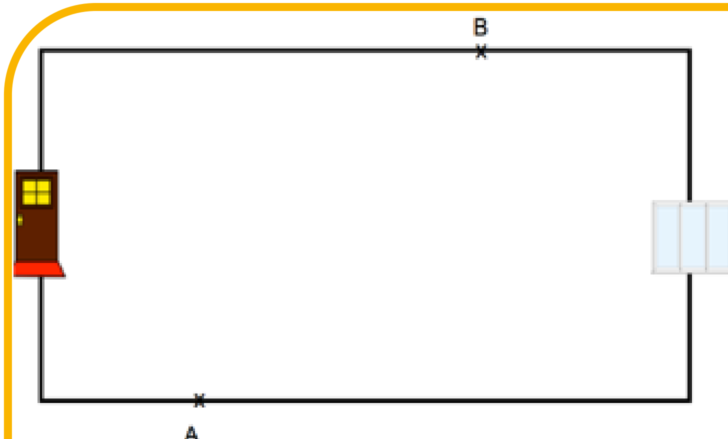
Shade the region that is:
- closer to A than B
- less than 4 cm from C

Line bisector of A and B

Circle with radius 4cm

Key Words

Bisect
Radius
Region
Shade



1cm = 1m

There are two burglar alarm sensors, one at A and one at B.

The range of each sensor is 4m.

The alarm is switched on.

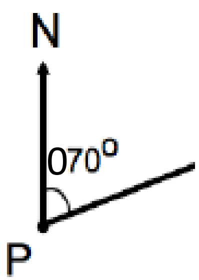
Is it possible to walk from the front door to the patio door without setting off the alarm?

Year 9 Higher
SCALES AND BEARINGS

Key Concepts

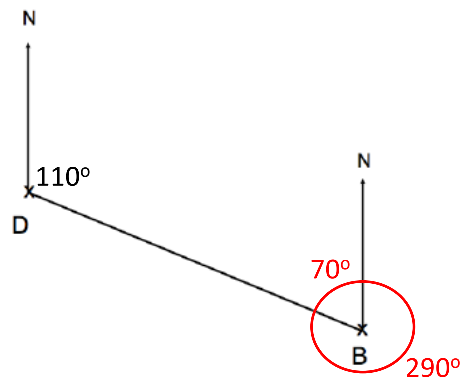
Scales are used to reduce real world dimensions to a useable size.

A **bearing** is an angle, measured **clockwise** from the **north** direction. It is given as a **3 digit** number.



Examples

The diagram shows the position of a boat B and dock D.



The scale of the diagram is 1cm to 5km.

a) Calculate the real distance between the boat and the dock.

$6cm = 6 \times 5$

$= 30km$

b) State the bearing of the boat from the dock.

110^o

c) Calculate the bearing of the dock from the dock.

$180^o - 110^o = 70^o$ because the angles are cointerior

$360^o - 70^o = 290^o$ because angles around a point equal 360°

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674-679,492-495

Key Words

Scale

Bearing

Clockwise

North

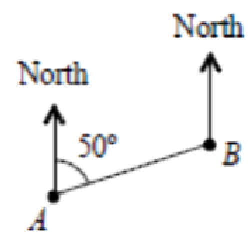
Links

Geography

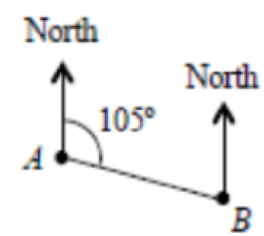
Find the bearing of A from B

(Diagrams not drawn to scale):

a)



b)



ANSWERS: (a) 230° (b) 285°

Year 9 Higher
PYTHAGORAS AND TRIGONOMETRY

Key Concepts

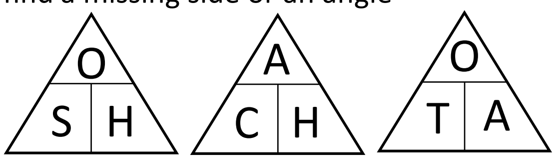
Pythagoras' theorem and basic trigonometry both work with **right angled triangles.**

Pythagoras' Theorem – used to find a missing length when two sides are known

$$a^2 + b^2 = c^2$$

c is always the hypotenuse (the longest side)

Basic trigonometry SOHCAHTOA – used to find a missing side or an angle

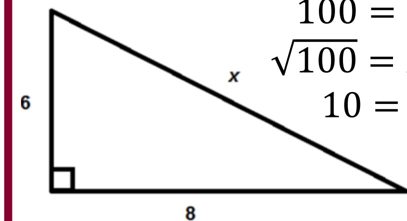


When finding the missing angle we must press **SHIFT** on our calculators first.

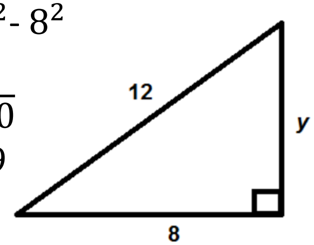
Examples

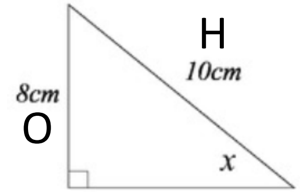
Pythagoras' Theorem

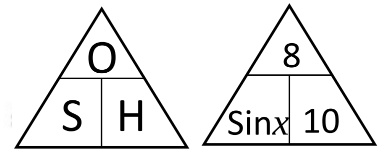
$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = x^2$$
$$100 = x^2$$
$$\sqrt{100} = x$$
$$10 = x$$



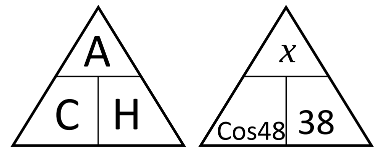
$$a^2 + b^2 = c^2$$
$$a^2 + 8^2 = 12^2$$
$$a^2 = 12^2 - 8^2$$
$$a^2 = 80$$
$$a = \sqrt{80}$$
$$a = 8.9$$



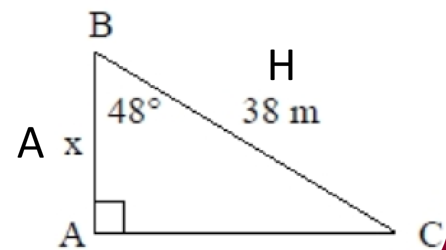




$$\sin x = \frac{8}{10}$$
$$x = \sin^{-1}\left(\frac{8}{10}\right)$$
$$x = 53.1^o$$



$$\cos 48 = \frac{x}{38}$$
$$38 \times \cos 48 = x$$
$$x = 25.4m$$



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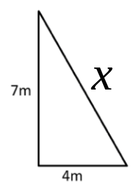
498-499, 509-515

Key Words

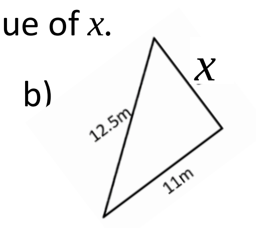
Right angled triangle
Hypotenuse
Opposite
Adjacent
Sine
Cosine
Tangent

Find the value of x.

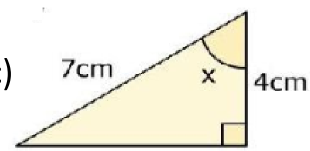
a)



b)



c)



d)

ANSWERS: a) 8.06m b) 5.94m c) 55.15° d) 2.34cm

Half Term 4

Year 9 Higher
AVERAGES FROM A TABLE

Key Concepts

Modal group (mode)
Group with the highest frequency

Median group
Find the cumulative frequency of the frequency. The median lies in the group which holds the $\frac{Total\ frequency+1}{2}$ number

Estimate the mean
From grouped data the mean can only be an estimate as we do not know where the data lies in each group.

$$\frac{Total\ fx}{Total\ f}$$

Examples

	Frequency (f)	Midpoint (x)	fx
$0 < x \leq 10$	10	5	50
$10 < x \leq 20$	15	15	225
$20 < x \leq 30$	23	25	575
$30 < x \leq 40$	7	35	245
Total	55		1095

a) Identify the modal group from this data set.

$20 < x \leq 30$

b) Identify the group in which the median would lie.

$$\frac{Total\ frequency + 1}{2} = \frac{56}{2} = 28th$$

Using the cumulative frequency of the groups the 28th lies in the groups $20 < x \leq 30$

c) Estimate the mean of this data:

$$\frac{Total\ fx}{Total\ f} = \frac{1095}{55} = 19.9$$

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414-418

Key Words
Midpoint
Mean
Median
Modal

Cost	Frequency	Midpoint	
$0 < c \leq 4$	2		
$4 < c \leq 8$	3		
$8 < c \leq 12$	5		
$12 < c \leq 16$	12		
$16 < c \leq 20$	3		

From the data:

a) Identify the modal group
b) Identify the group which holds the median
c) Estimate the mean

ANSWERS: a) $12 < c \leq 16$ b) 13th value is in the group $12 < c \leq 16$ c) $\frac{294}{25} = 11.76$

Year 9 Higher
STATISTICAL DIAGRAMS

Key Concepts

A **frequency polygon** is a line graph which connects the midpoints of grouped data.

A **pie chart** represents data into proportional sections.

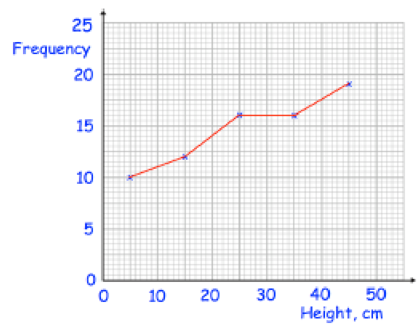
A **scatter-graph** shows the relationship between two variables. **Correlation** is used to describe the relationships.

Positive Correlation | Negative Correlation | No Correlation

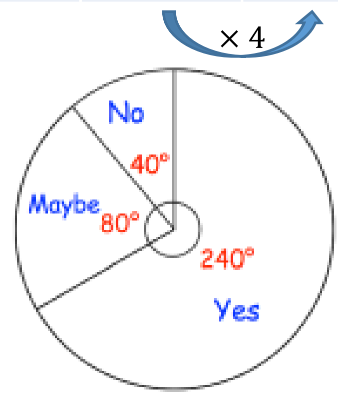
Examples

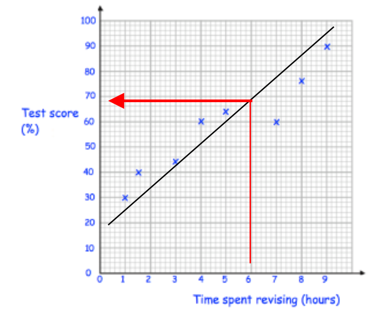
Plot at the midpoint

Length, cm	Frequency
$0 < x \leq 10$	10
$10 < x \leq 20$	12
$20 < x \leq 30$	16
$30 < x \leq 40$	16
$40 < x \leq 50$	19



Answer	Frequency	Angle
Yes	60	240
No	10	40
Maybe	20	80
Total	90	360





a) What type of correlation is shown?
Positive correlation

b) Another student spent 6 hours revising for the test. Find an estimate of their test score.
Draw a line of best fit and read from it - 68%

c) Explain why it might not be sensible to use the scatter graph to estimate the score for a student that spent 15 hours revising.
It is out of the data range.

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441,427-429, 453-454

Key Words

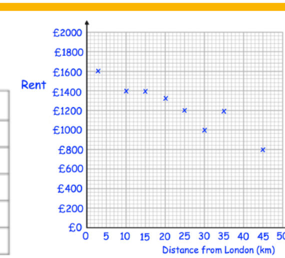
Midpoint
Frequency polygon
Pie chart
Degrees
Scatter graph
Correlation
Line of best fit

1) Draw a frequency polygon using this data.

Marks	Frequency
$0 < m \leq 10$	8
$10 < m \leq 20$	11
$20 < m \leq 30$	23
$30 < m \leq 40$	19
$40 < m \leq 50$	15

2) Draw a pie chart using this data.

Make	Frequency
Ford	8
Mazda	14
Volkswagen	21
Fiat	20
Honda	9



3a) What type of correlation is shown?

b) The distance from London of a house is 22km. What is an estimate of the rent it will cost?

ANSWERS: 2) Angles – 40, 70, 105, 100, 45 3a) Negative correlation b) Between £1200 and £1300

Year 9 Higher
CUMULATIVE FREQUENCY AND BOX PLOTS

Key Concepts

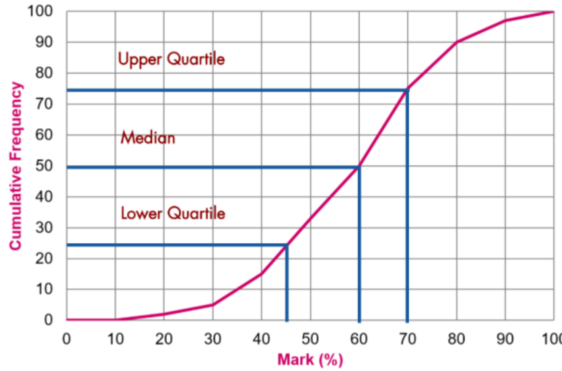
A **cumulative frequency** graph shows a running total of frequency.

We can read the **median** and the **interquartile range** from this graph.

A **box plot** shows the distribution of data using **minimum, maximum, median** and **quartiles**.

Mark	Freq	CF
$0 < x \leq 10$	0	0
$10 < x \leq 20$	4	4
$20 < x \leq 30$	1	5
$30 < x \leq 40$	10	15
$40 < x \leq 50$	17	32
$50 < x \leq 60$	18	50
$60 < x \leq 70$	24	74
$70 < x \leq 80$	16	90
$80 < x \leq 90$	6	96
$90 < x \leq 100$	4	100

Plot at the upper bound



Median and quartiles are found from the y axis:

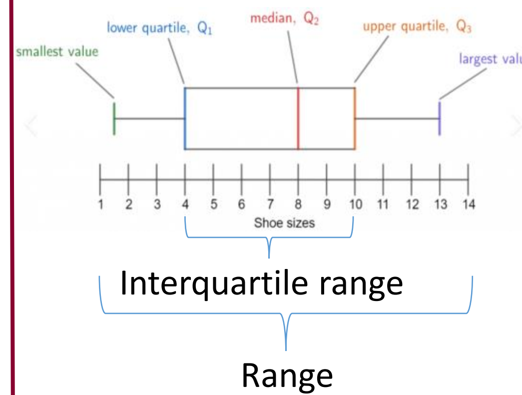
Lower quartile = 25% of the way through the data
= 45

Median = 50% of the way through the data
= 60

Upper quartile = 75% of the way through the data
= 70

Interquartile range = UQ – LQ
= 70 – 45
= 25

Examples



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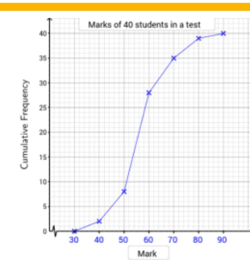
434-440

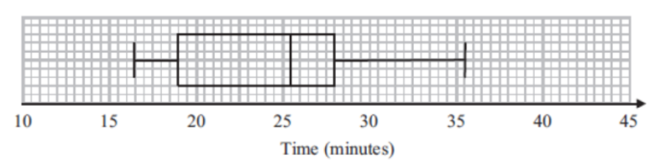
Key Words

Cumulative frequency
Box plot
Range
Interquartile range
Median
Quartiles
Minimum/maximum values

1) Read from the cumulative frequency graph to find the median and the interquartile range.



2) Read from the box plot the median, range and interquartile range.

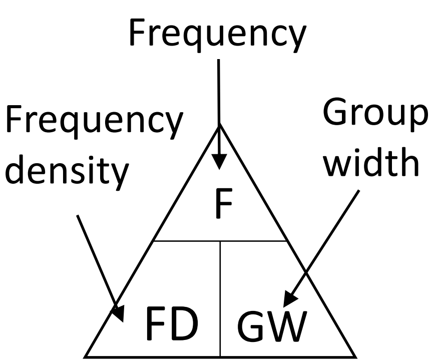


ANSWERS: 1) Median = 56, Interquartile range = 64 – 52 = 12 2) Median = 26, Range = 35.5 – 16.5 = 19, Interquartile range = 28 – 19 = 9

Year 9 Higher
HISTOGRAMS

Key Concepts

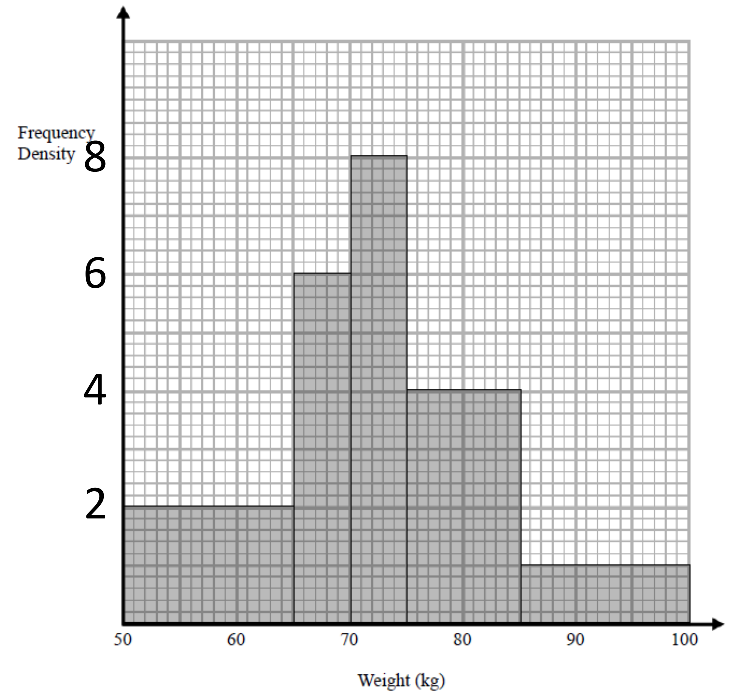
A **Histogram** is a graphical representation of data consisting of rectangles whose **area is proportional to the frequency** of a variable and whose **width is equal to the group width**.



Example

A group of people are weighed and their results recorded. Below is their data.
A histogram is used to represent this data.

Weight	Frequency	Frequency density
50 < w ≤ 65	30	30 ÷ 15 = 2
65 < w ≤ 70	30	30 ÷ 5 = 6
70 < w ≤ 75	40	40 ÷ 5 = 8
75 < w ≤ 85	40	40 ÷ 10 = 4
85 < w ≤ 100	15	15 ÷ 15 = 1



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443-449

Key Words
Histogram
Frequency density
Group width
Median

Speed (mph)	Frequency
40 < s ≤ 55	6
55 < s ≤ 60	10
60 < s ≤ 65	46
65 < s ≤ 75	48
75 < s ≤ 90	6

Calculate the frequency density for this table of information.

On a separate set of axes, draw your histogram.

ANSWERS: Frequency densities = 0.4, 2, 9.2, 4.8, 0.4

Half Term 5

Year 9 Higher

TWO WAY TABLES AND PROBABILITY TABLES

Key Concepts

Two way tables are used to tabulate a number of pieces of information.

Probabilities can be formulated easily from two way tables.

Probabilities can be written as a **fraction, decimal or a percentage** however we often work with fractions. You do not need to simplify your fractions in probabilities.

Estimating the number of times an event will occur

Probability × no. of trials

Examples

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	$3x$	x-5	$2x$

A counter is chosen at random, the probability it is red is $\frac{9}{100}$. Work out the probability is black.

$$9 + 3x + x - 5 + 2x = 100$$
$$6x + 4 = 100$$
$$x = 16$$

Number of black counters = 16 – 5

= 11

Probability of choosing black = $\frac{11}{100}$

80 children went on a school trip. They went to London or to York.

23 boys and 19 girls went to London. 14 boys went to York.

	London	York	Total
Girls	19	**24**	**43**
Boys	23	14	**37**
Total	**42**	**38**	80

What is the probability that a person is chosen that went to London? $\frac{42}{80}$

If a girl is chosen, what is the probability that she went to York? $\frac{24}{38}$

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353, 422-424

Key Words

Two way table

Probability

Fraction

Outcomes

Frequency

	1	2	3
Prob	0.37	$2x$	x

1a) Calculate the probability of choosing a 2 or a 3.

b) Estimate the number of times a 2 will be chosen if the experiment is repeated 300 times.

2a) Complete the two way table:

	Year Group			Total
	9	10	11	
Boys			125	407
Girls		123		
Total	303	256		831

b) What is the probability that a Y10 is chosen, given that they are a girl .

ANSWERS: 1) a) P(2) = 0.42 P(3) = 0.21 b) 0.42×300=126 2b) 123/424

Year 9 Higher
VENN DIAGRAMS

Key Concepts

Venn diagrams show all possible relationships between different sets of data.

Probabilities can be derived from Venn diagrams. Specific notation is used for this:

$P(A \cap B)$ = Probability of A **and** B

$P(A \cup B)$ = Probability of A **or** B

$P(A')$ = Probability of **not** A

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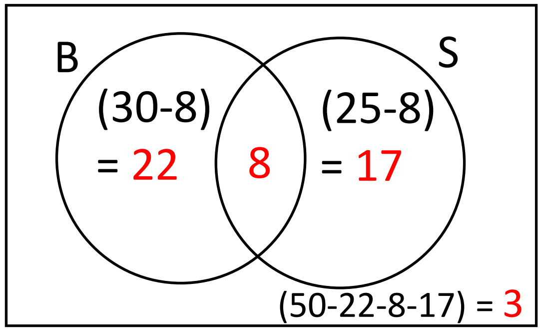
372-388, 391

Key Words
Venn diagram
Union
Intersection
Probability
Outcomes

Example

Out of 50 people surveyed:

30 have a brother
25 have a sister
8 have both a brother and sister



a) Complete the Venn diagram

b) Calculate:

i) $P(A \cap B)$ $= \frac{8}{50}$ ii) $P(A \cup B)$ $= \frac{47}{50}$ iii) $P(B')$ $= \frac{20}{50}$

iv) The probability that a person with a sister, does not have a brother.

$$= \frac{8}{25}$$

40 students were surveyed:

20 have visited France
15 have visited Spain
10 have visited both France and Spain

a) Complete a Venn diagram to represent this information.

b) Calculate:

i) $P(F \cap S)$ ii) $P(F \cup S)$ iii) $P(S')$

iv) The probability someone who has visited France, has not gone to Spain.

ANSWERS: bi) 10/40 ii) 25/40 iii) 25/40 iv) 20/40 = 1/2

Year 9 Higher
PROBABILITY TREE DIAGRAMS

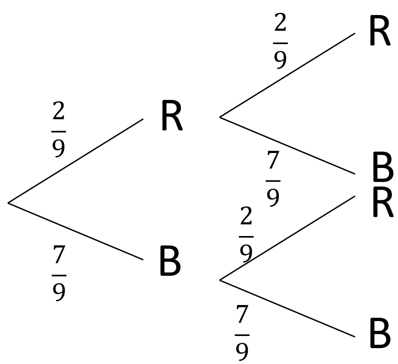
Key Concepts

Independent events are events which do not affect one another.

Dependent events affect one another's probabilities. This is also known as **conditional probability**.

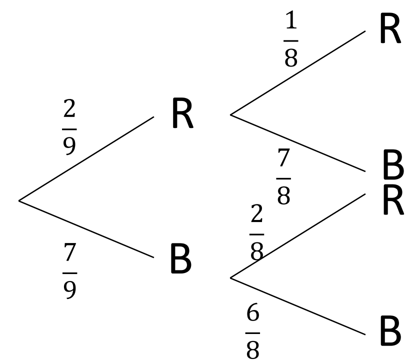
Examples

There are red and blue counters in a bag.
The probability that a red counter is chosen is $\frac{2}{9}$.
A counter is chosen and **replaced**, then a second counter is chosen.
Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



Prob of two reds:
$$\frac{2}{9}\times\frac{2}{9}=\frac{4}{81}$$
Prob of two blues :
$$\frac{7}{9}\times\frac{7}{9}=\frac{49}{81}$$
Prob of same colours:
$$\frac{4}{81}+\frac{49}{81}=\frac{53}{81}$$

There are red and blue counters in a bag.
The probability that a red counter is chosen is $\frac{2}{9}$.
A counter is chosen and **not replaced**, then a second counter is chosen.
Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



Prob of two reds:
$$\frac{2}{9}\times\frac{1}{8}=\frac{2}{72}$$
Prob of two blues :
$$\frac{7}{9}\times\frac{6}{8}=\frac{42}{72}$$
Prob of same colours:
$$\frac{2}{72}+\frac{42}{72}=\frac{44}{72}$$

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361-362, 364-367, 389-390

Key Words
Independent
Dependant
Conditional
Probability
Fraction

1) There are blue and green pens in a drawer. There are 4 blues and 7 greens.
A pen is chosen and then **replaced**, then a second pen is chosen.
Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

2) There are blue and green pens in a drawer. There are 4 blues and 7 greens.
A pen is chosen and **not replaced**, then a second pen is chosen.
Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

ANSWERS: 1) 56/121 2) 56/110

Half Term 6

Year 9 Higher
TYPES OF ANGLE AND ANGLES IN POLYGONS

Key Concepts

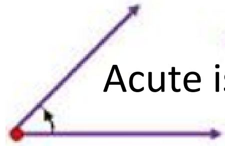
Regular polygons have equal lengths of sides and equal angles.

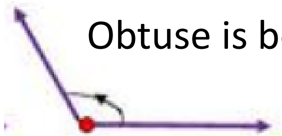
Angles in polygons
Sum of interior angles
$= (number\ of\ sides - 2) \times 180$

Exterior angles of **regular** polygons = $\frac{360}{number\ of\ sides}$

Types of angle
There are four types which need to be identified – acute, obtuse, reflex and right angled.

Examples

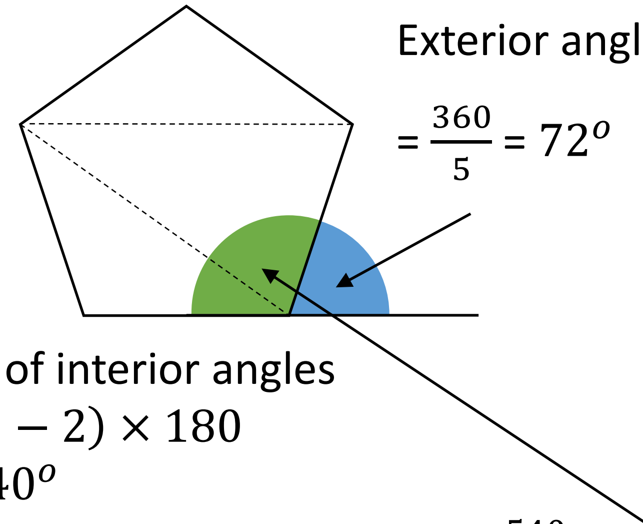

Acute is less than 90°


Obtuse is between 90° and 180°


Right angled is 90°


Reflex is between 180° and 360°

Regular Pentagon



Exterior angles

$= \frac{360}{5} = 72^o$

Sum of interior angles
$= (5 - 2) \times 180$
$= 540^o$

Interior angle = $\frac{540}{5} = 108^o$

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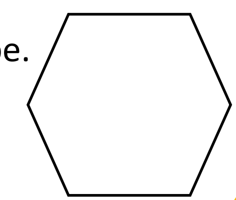
455, 456, 560-564

Key Words
Polygon
Interior angle
Exterior angle
Acute
Obtuse
Right angle
Reflex

Questions
1) Calculate the sum of the interior angles for this regular shape.
2) Calculate the exterior angle for this regular shape.
3) Calculate the size of one interior angle in this regular shape.



ANSWERS: 1) 720° 2) 60° 3) 120°

Year 9 Higher

ANGLE FACTS INCLUDING ON PARALLEL LINES

Key Concepts

Angles in a **triangle equal 180º**.

Angles in a **quadrilateral equal 360º**.

Vertically opposite angles are equal in size.

Angles on a **straight line equal 180º**.

Base angles in an isosceles triangle are equal.

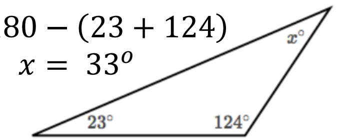
Alternate angles are equal in size.

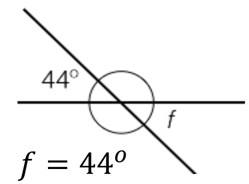
Corresponding angles are equal in size.

Allied/co-interior angles are equal 180º.

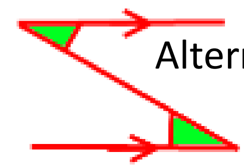
Examples

$x = 180 - (23 + 124)$
$x = 33^o$

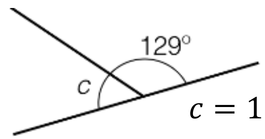




$f = 44^o$

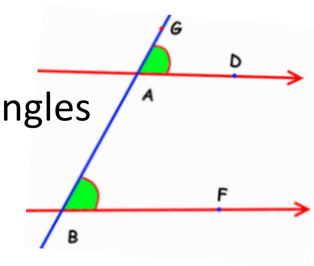


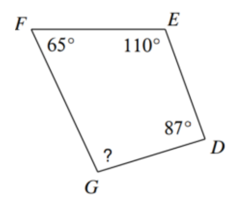
Alternate angles are equal



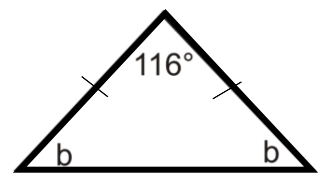
$c = 180 - 129$
$x = 51^o$

Corresponding angles are equal

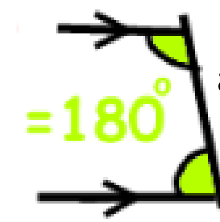




$? = 360 - (65 + 110 + 87)$
$? = 98^o$



$b = (180 - 116) \div 2$
$b = 32^o$



Allied/co-interior angles equal 180º

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477-480, 481-483

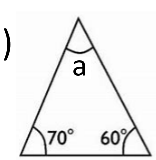
Key Words

Angle
Vertically opposite
Straight line
Alternate
Corresponding
Allied
Co-interior

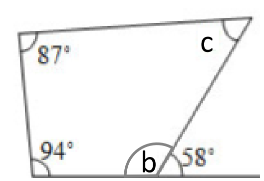
Questions

Calculate the missing angle:

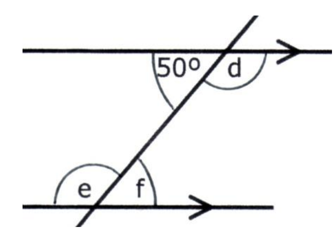
a)



b)



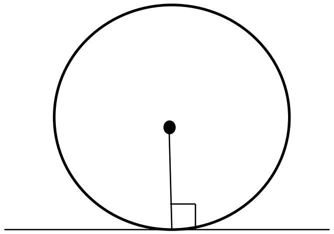
c)



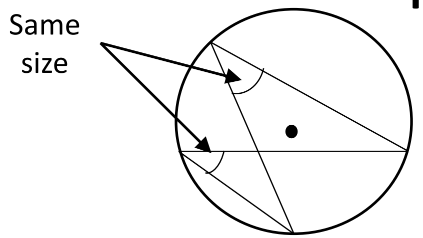
ANSWERS: 1) a=50º 2) b=122º c=57º 3) d=130º e=130º f=50º

Year 9 Higher
CIRCLE THEOREMS

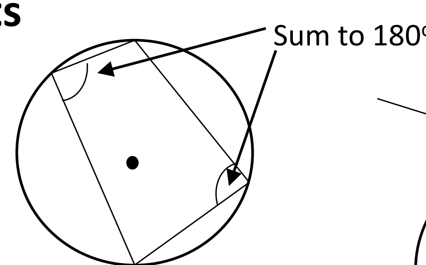
Key Concepts



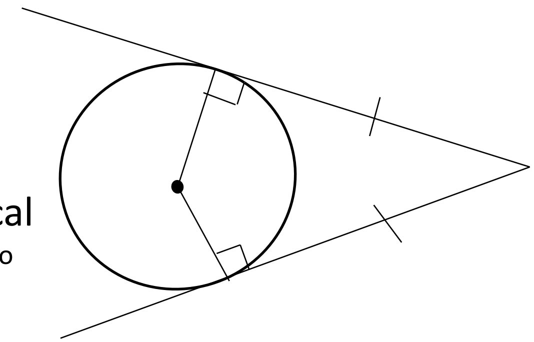
The angle between a radius and a tangent is 90°



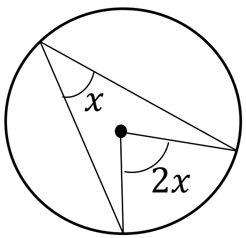
Angles at the circumference are equal



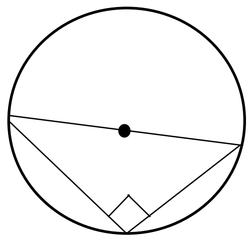
Opposite angles in a cyclical quadrilateral sum to 180°



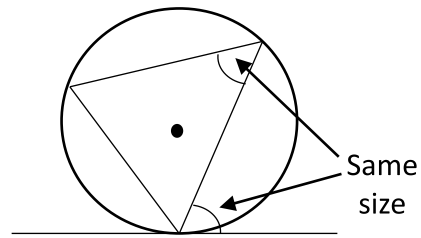
From any point you can only draw two tangents, and they are equal in length



The angle at the centre is twice that at the circumference



The angle in a semi circle is 90°



The alternate segment theorem

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593-606

Key Words

Radius
Centre
Tangent
Circumference
Right angle

Try look, cover, write, check to be able to identify and describe each of the 7 circle theorems.

1. Read through the theorems
2. Cover them over
3. Attempt to recreate them on another sheet of paper
4. Check how many you remembered perfectly. Try again until you have all 7.

ANSWER