Year 9 Maths Knowledge Organiser (F)

Half Term 1

Year 9 Foundation

INTEGERS, ROUNDING AND PLACE VALUE

Key Concepts

Digits are the individual components of a number.

Integers are whole numbers.

Rounding rules:
A value of 5 to 9 rounds the number up.
A value of 0 to 4 keeps the number the same.

Examples

Order the following numbers starting with the smallest:

1) 5, -3, 4, 7, -2
-3, -2, 4, 5, 7

2) 0.067 0.6 0.56 0.65 0.605
Rewrite 0.067, 0.600, 0.560, 0.650, 0.605
0.067 0.56 0.6 0.605 0.65

Round 3.527 to:

a) 1 decimal place
3 . 5 2 7 → 3.5

b) 2 decimal places
3 . 5 2 7 → 3.53

c) 1 significant figure
3 . 5 2 7 → 4

hegartymaths

1 – 3, 31 – 32

Key Words

Integer	Even
Digit	Odd
Decimal place	
Significant figures	

A) Order the following numbers starting with the smallest:
1) 6, -2, 0, -5, 3 2) 0.72, 0.7, 0.072, 0.07, 0.702

B) Round the following numbers to the given degree of accuracy
1) 14. 1732 (1 d.p.) 2) 0.0568 (2 d.p.) 3)3418 (1 S.F)

ANSWERS: A1) -5, -2, 0, 3, 6 2) 0.07, 0.072, 0.7, 0.702, 0.72
B1) 14.2 2) 0.06 3) 3000

Year 9 Foundation
DECIMALS

Key concepts

Place value:

Th H T U . t h th

When adding and subtracting decimals we must ensure the decimal places are underneath each other when setting up.

When multiplying decimals, calculate without the decimal point and use estimation to help replace it.

Examples

345.461

3 hundreds

Forty or 4 tens

5 units

4 tenths

6 hundredths

1 thousandths

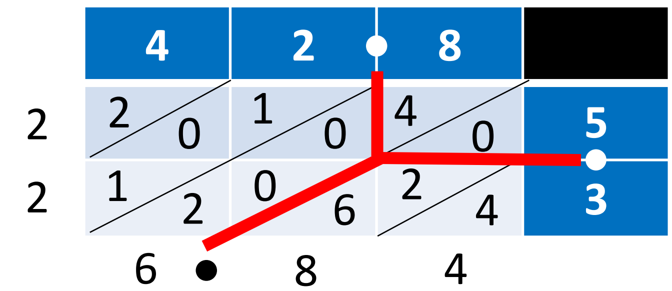
42.8 + 5.32

$$\begin{array}{r} 42.8\color{red}{0} \\ +\ 5.32 \\ \hline 48.12 \end{array}$$

42.8 − 5.32

$$\begin{array}{r} \overset{3}{\cancel{4}}\,{}^{1}2.\overset{7}{\cancel{8}}\,{}^{1}\color{red}{0} \\ -\ \ 5.32 \\ \hline 37.48 \end{array}$$

42.8 × 5.3



2 2 6 . 8 4

Estimated answer 40 × 5 = 200

hegartymaths

102 – 110

Key Words

Decimal

Tenths

Hundredths

Thousandths

A) What is the value of the 4 in each number?

1) 498 2) 8746 3) 6.243 4) 1.004

B) Work out:

1) 3.1 + 5.27 2) 16.4 – 9.18 3) 0.03 × 500 4) 3.4 × 5.6

5) 4.79 × 6.8

ANSWERS: A 1) 4 hundred 2) forty 3) 4 hundredths 4) 4 thousandths

B 1) 8.37 2) 7.22 3) 15 4) 19.04 5) 32.572

Year 9 Foundation
FRACTIONS, DECIMALS AND PERCENTAGES

Key Concepts

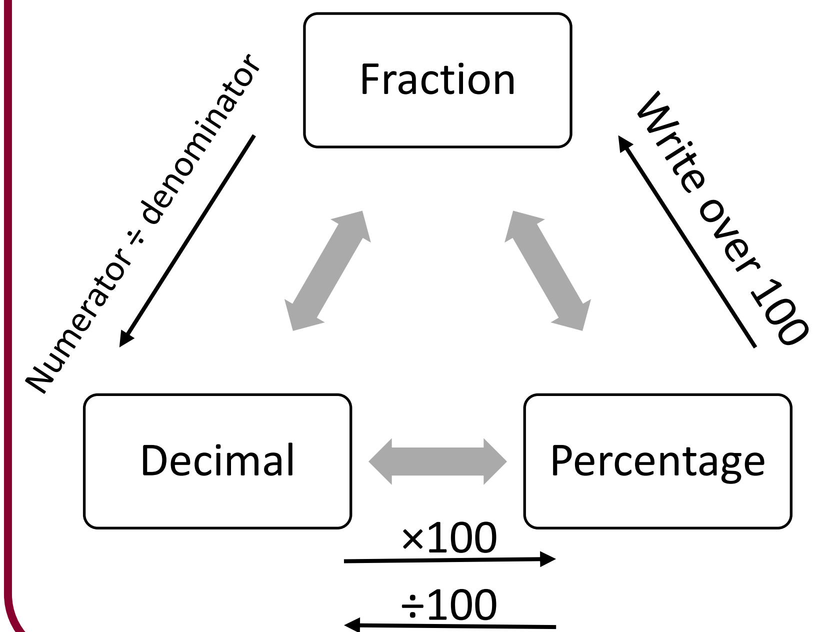
A **fraction** is a numerical quantity that is not a whole number.

A **decimal** is a number written using a system of counting based on the number 10.

Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths
8	7	6	5	.	4	3	2

A **percentage** is an amount out of 100.

Examples



Order the following in ascending order:

$\frac{3}{5}$ 62% 0.67 $\frac{7}{10}$ 0.665

$\frac{3}{5} \xrightarrow{\times 20} \frac{60}{100} \rightarrow 60\%$

$62\% \rightarrow 62\%$

$0.67 \xrightarrow{\times 100} 67\%$

$\frac{7}{10} \xrightarrow{\times 10} \frac{70}{100} \rightarrow 70\%$

$0.665 \xrightarrow{\times 100} 66.5\%$

60% 62% 67% 70% 66.5%

$\frac{3}{5}$ 62% 0.665 0.67 $\frac{7}{10}$

hegartymaths

73-76, 82-83

Key Words

Fraction
Decimal
Percentage
Division
Multiply

1) Convert the following into percentages:
   a) 0.4 b) 0.08 c) $\frac{6}{20}$ d) $\frac{3}{25}$
2) Compare and order the following in ascending order:

   $\frac{3}{4}$ 76% 0.72 $\frac{4}{5}$ 0.706

ANSWERS 1a) 40% b) 8% c) 30% d) 12% 2) 0.706 0.72 $\frac{3}{4}$ 76% $\frac{4}{5}$

Year 9 Foundation

INDICES AND ROOTS

Key Concepts

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m}$$

Examples

Simplify each of the following:

1) $a^6 \times a^4 = a^{6+4}$
$= a^{10}$

2) $3^6 \times 3^5 = 3^{6+5}$
$= 3^{11}$

3) $a^6 \div a^4 = a^{6-4}$
$= a^2$

4) $9^6 \div 9^3 = 9^{6-3}$
$= 9^3$

5) $(a^6)^4 = a^{6\times 4}$
$= a^{24}$

6) $(3a^4)^3 = 3^3 a^{4\times 3}$
$= 27a^{12}$

7) $a^{-1} = \frac{1}{a^1}$

8) $a^{-2} = \frac{1}{a^2}$

9) $a^{\frac{1}{2}} = \sqrt[2]{a^1} = \sqrt{a}$

hegartymaths

102 – 110

Key Words

Powers
Roots
Indices
Reciprocal

Write as a single power: 1) $a^3 \times a^2$ 2) $b^4 \times b$ 3) $d^{-5} \times d^{-1}$ 4) $m^6 \div m^2$
5) $n^4 \div n^4$ 6) $\frac{8^4 \times 8^5}{8^6}$ 7) $\frac{4^9 \times 4}{4^3}$

Evaluate : 1) $(3^2)^5$ 2) 2^{-2} 3) $81^{\frac{1}{2}}$ 4) $27^{\frac{1}{3}}$

ANSWERS: 1) a^5 2) b^5 3) d^{-6} 4) m^4 5) 1 6) 8^3 7) 4^7
1) 3^{10} 2) $\frac{1}{4}$ 3) 9 4) 3

Year 9 Foundation
FACTORS, MULTIPLES AND PRIMES

Key Concepts

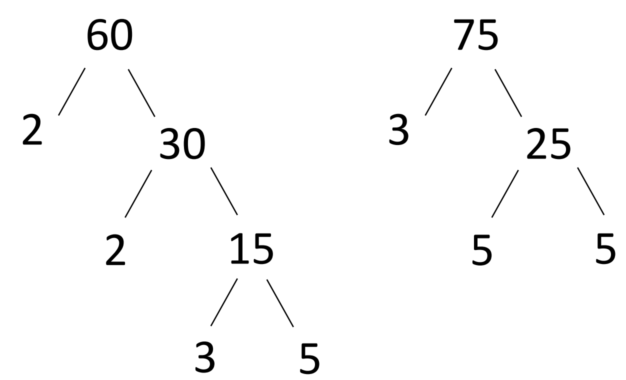
Prime factor decomposition
Breaking down a number into its prime factors

Highest common factor
Finding the largest number which divides into all numbers given

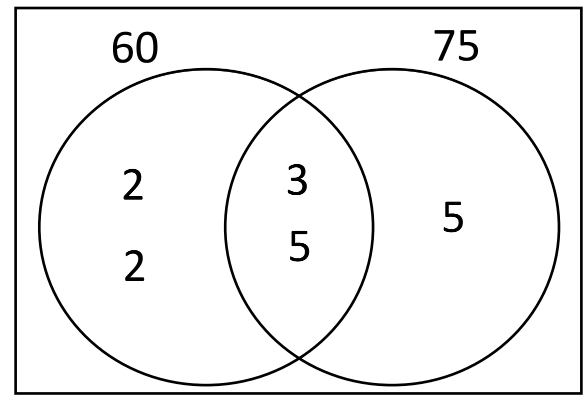
Lowest common multiple
Finding the smallest number which both numbers divide into

Examples

Find the **highest common factor** and **lowest common multiple** of 60 and 75:



$$2 \times 2 \times 3 \times 5 \qquad 3 \times 5 \times 5$$
$$2^2 \times 3 \times 5 \qquad 3 \times 5^2$$



HCF – Mulitiply all numbers in the intersection
$= 3 \times 5 = 15$

LCM – Multiply all numbers in the Venn diagram
$= 2 \times 2 \times 3 \times 5 \times 5 = 300$

hegartymaths

29 – 32,34,35

Key Words

Factor
Multiple
Prime
Highest Common Factor
Lowest Common Multiple

Questions

1) Write 80 as a product of its prime factors
2) Write 48 as a product of its prime factors
3) Find the LCM and HCF of 80 and 48

ANSWERS: 1) $2^4 \times 5$ 2) $2^4 \times 3$ 3) LCM = 240 and HCF = 16

Half Term 2

Year 9 Foundation

EXPRESSIONS/EQUATIONS/IDENTITIES AND SUBSTITUTION

Key Concepts

A **formula** involves two or more letters, where one letter equals an **expression** of other letters.

An **expression** is a sentence in algebra that does NOT have an equals sign.

An **identity** is where one side is the equivalent to the other side.

When **substituting** a number into an expression, replace the letter with the given value.

Examples

1) $5(y + 6) \equiv 5y + 30$ is an identity as when the brackets are expanded we get the answer on the right hand side

2) $5m - 7$ is an expression since there is no equals sign

3) $3x - 6 = 12$ is an equation as it can be solved to give a solution

4) $C = \frac{5(F - 32)}{9}$ is a formula (involves more than one letter and includes an equal sign)

5) Find the value of $3x + 2$ when $x = 5$

$(3 \times 5) + 2 = 17$

6) Where $A = b^2 + c$, find A when $b = 2$ and $c = 3$

$A = 2^2 + 3$

$A = 4 + 3$

$A = 7$

hegartymaths

153, 154, 189, 287

Key Words

Substitute

Equation

Formula

Identity

Expression

Questions

1) Identify the equation, expression, identity, formula from the list (a) $v = u + at$ (b) $u^2 - 2as$ (c) $4x(x - 2) = x^2 - 8x$ (d) $5b - 2 = 13$

2) Find the value of $5x - 7$ when $x = 3$

3) Where $A = d^2 + e$, find A when $d = 5$ and $e = 2$

ANSWERS: 1) (a) formula (b) expression (c) identity (d) equation 2) 8 3) $A = 27$

Year 9 Foundation
ALGEBRAIC EXPRESSIONS

Key Concepts

When collecting like terms involving addition or subtraction, add/subtract the numbers in front of the letters.

If the like terms are multiplied, multiply the numbers in front of the letters and put the letters next to each other.

If the like terms are divided, divide the numbers in front of the letters.

Examples

Simplify the following expressions:

1) $4p + 6t + p - 2t = 5p + 4t$
2) $3 + 2t + p - t + 2 = 5 + t + p$
3) $f + 3g - 4f = 3g - 3g$
4) $f^2 + 4f^2 - 2f^2 = 3f^2$
5) $6a \times 3b \times 2c = 36abc$
6) $\frac{9b}{3} = 3b$

hegartymaths

151 – 152, 156 – 157

Key Words

Simplify
Term
Collect

Questions

Simplify:

1) $7p + 3q + p - 3q$
2) $5 + 4t + 3p - 2t + 7$
3) $m - 8g - 5m$
4) $b^2 - 7b^2 + 2b^2$
5) $2a \times 5b \times 4c$
6) $8m \times 3n \times 2m$
7) $\frac{36p}{12}$
8) $\frac{6t}{18}$

ANSWERS: 1) 8p 2) $12 + 2t + 3p$ 3) $-4m - 8g$ 4) $-4b^2$ 5) 40abc 6) $48m^2n$ 7) 3p 8) $\frac{t}{3}$

Year 9 Foundation
EXPAND AND SIMPLIFY BRACKETS

Key Concepts

Expanding brackets
Single: Where each term inside the bracket is multiplied by the term on the outside of the bracket.
Double: Where each term in the first bracket is multiplied by all terms in the second bracket.

Factorising expressions
Putting an expression back into brackets. To "factorise fully" means take out the HCF.

Difference of two squares
When two brackets are repeated with the exception of a sign change. All numbers in the original expression will be square numbers.

Examples

Linear expressions

Expand and simplify where appropriate

1) 7 (3 + a) = 21 + 7a

2) 2(5 + a) + 3(2 + a) = 10 + 2a + 6 + 3a
= 5a + 16

3) Factorise $9x + 18$ = $9(x + 2)$

4) Factorise $6e^2 - 3e$ = $3e(2e - 1)$

Quadratic expressions

Expand and simplify:

1) $(p + 2)(2p - 1)$
$= 2p^2 + 4p - p - 2$
$= 2p^2 + 3p - 2$

2) $(p + 2)^2$
$(p + 2)\,(p + 2)$
$= p^2 + 2p + 2p + 4$
$= p^2 + 4p + 4$

Factorise:

3) $x^2 - 2x - 3$
$= (x - 3)(x + 1)$

Factorise and solve:

4) $x^2 + 4x - 5 = 0$
$(x - 1)(x + 5) = 0$
Therefore the solutions are:
Either $x - 1 = 0$
$x = 1$
Or $x + 5 = 0$
$x = -5$

hegartymaths
160, 162-164, 168-169, 223-228, 230-234

Key Words
Expand
Factorise
Simplify
Product
Solve

1) Expand and simplify (a) 3(2 – 7f) (b) 5(m – 2) + 6 (c) 3(4 + t) + 2(5 + t)

2) Factorise (a) 6m + 12t (b) 9t – 3p (c) $4d^2 - 2d$

3) Expand (5g – 4)(2g + 1)

4) (a) **Factorise** $x^2 - 8x + 15$ **(b) Factorise and solve** $x^2 + 7x + 10 = 0$

ANSWERS: 1) (a) 6 – 21f (b) 5m – 4 (c) 22 + 5t 2) (a) 6(m + 2t) (b) 3(3t – p) (c) 2d(2d – 1) 3) $10g^2 - 3g - 4$ 4) (a) $(x - 3)(x - 5)$ (b) $x = -2$ or $x = -5$

Year 9 Foundation

REARRANGE AND SOLVE EQUATIONS

Key Concepts

Solving equations:
Working with inverse operations to find the value of a variable.

Rearranging an equation:
Working with inverse operations to isolate a highlighted variable.

In solving and rearranging we **undo the operations** starting from the last one.

Examples

For each step in solving an equation we must do the **inverse** operation

Solve:

$5(x - 3) = 20$

Expand

$5x - 15 = 20$

+15 +15

$5x = 35$

÷5 ÷5

$x = 7$

Solve:

$12 = 3x - 18$

+18 +18

$30 = 3x$

÷3 ÷3

$x = 10$

Solve:

$7p - 5 = 3p + 3$

-3p -3p

$4p - 5 = 3$

+5 +5

$4p = 8$

÷ 2 ÷ 2

$p = 2$

Rearrange to make r the subject of the formulae :

$$Q = \frac{2r - 7}{3}$$

×3 × 3

$$3Q = 2r - 7$$

+7 +7

$$3Q + 7 = 2r$$

÷ 2 ÷ 2

$$\frac{3Q + 7}{2} = r$$

hegartymaths
177-186, 280-284, 287

Key Words
Solve
Rearrange
Term
Inverse
operation

1) Solve $7(x + 2) = 35$
2) Solve $4x - 12 = 28$
3) Solve $4x - 12 = 2x + 20$
4) Rearrange to make x the subject:
$$y = \frac{3x + 4}{2}$$

ANSWERS: 1) $x = 3$ 2) $x = 10$ 3) $x = 16$ 4) $x = \frac{2y - 4}{3}$

Year 9 Foundation
EQUATIONS IN CONTEXT

Key Concepts

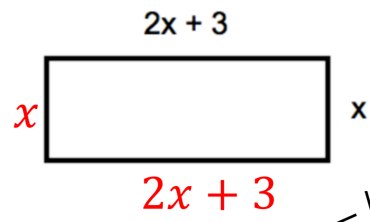
Algebra can be used to support us to find unknowns in a **contextual problem**.

We can always apply a letter to an unknown quantity, to then **set up an equation**.

It will often be used in area and perimeter problems and angle problems in geometry.

Solve to find the value of x when the perimeter is 42cm.

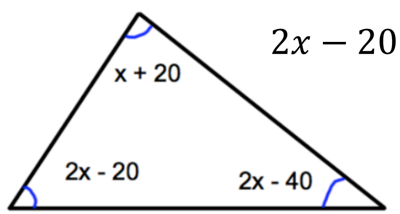
HINT: Write on all of the lengths of the sides.



We know the perimeter is 42cm

$$2x+3+2x+3+x+x=42$$
$$9x+6=42$$
$$6x=36$$
$$x=6$$

Angles in a triangle sum to 180º



$$2x-20+x+20+2x-40=180$$
$$5x-40=180$$
$$5x=220$$
$$x=45$$

Examples

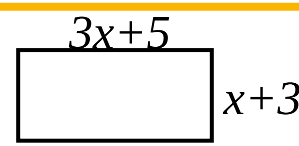
Jane is 4 years older than Tom.
David is twice as old as Jane.
The sum of their ages is 60.
Using algebra, find the age of each person.

Tom = x → 12
Jane = $x+4$ → $12+4=16$
David = $2x+8$ → $(2\times 12)+8=32$

$$x+x+4+2x+8=60$$
$$4x+12=60$$
$$4x=48$$
$$x=12$$

hegartymaths

Key Words
Solve
Term
Inverse
operation



1) If the perimeter is 40cm. What is the length of the longest side?

2) Jane is 12 years older than Jack.
Sarah is 3 years younger than Jack.
The sum of their ages is 36.
Using algebra, find the age of each person.

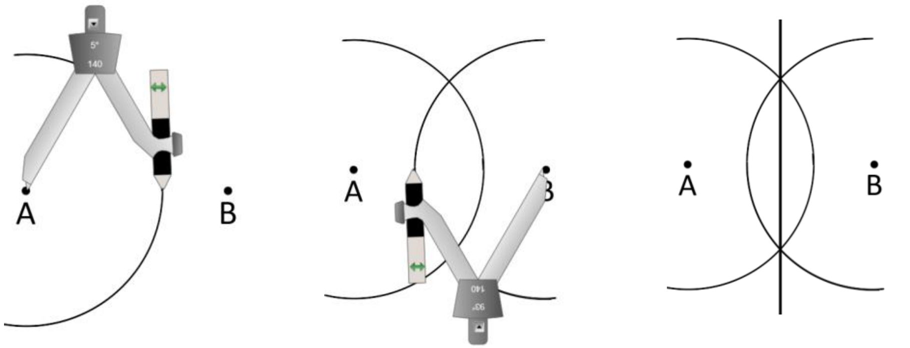
ANSWERS: 1) x = 3 therefore the longest length is 14cm 2) Jack = 9, *Jane* = 21, *Sarah* =6

Half Term 3

Year 9 Foundation
CONSTRUCTIONS

Examples

Bisect the distance between two points.

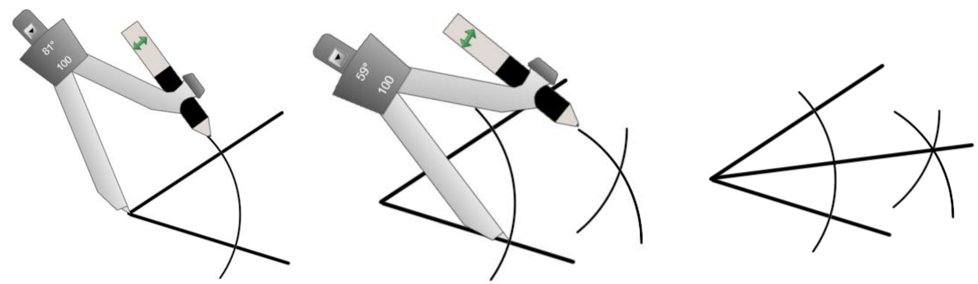


1) Open your compasses past halfway between the two points and draw an arc.

2) Keep your compasses at the same width and repeat from the other point.

3) Draw a line joining the two points where the arcs cross

Bisect an angle.



1) Open your compasses and draw an arc over both lines from the angle

2) Keep your compasses at the same width and draw two further arcs with the point of your compasses at the intersections.

3) Draw a line joining the two points where the arcs cross and the angle point

hegartymaths

660-661

Key Words

Compass
Bisect
Angle
Arc

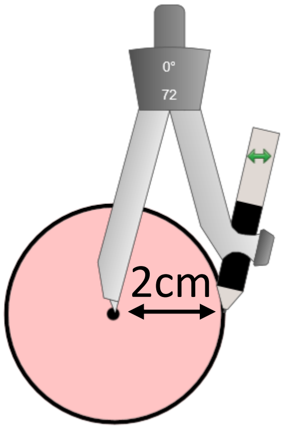
Try and recreate the above two constructions on paper using a pair of compasses and a pencil and ruler.

ANSWERS

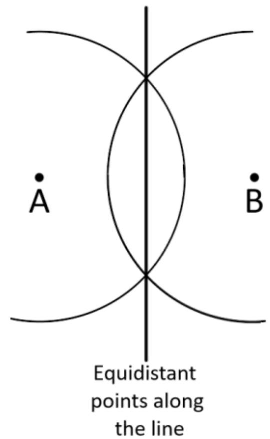
Year 9 Foundation
LOCI

Examples

Shading a **region** within 2cm from a **given point**.

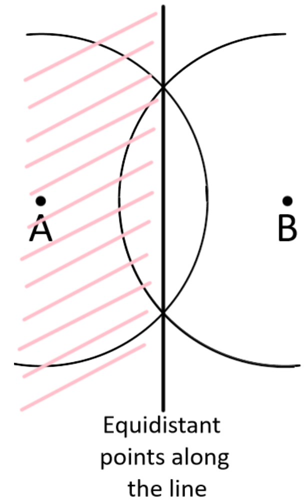


Find where a point can be **equidistant** from two others.



Use your skills from constructions and complete the **perpendicular bisector**.

Shading a **region** which is **closer to point A than point B**.



Use your skills from constructions and complete the **perpendicular bisector**. Then shade in the side of the line closer to the given point.

hegartymaths

674-679

Key Words

Compass
Bisect
Shade
Region
Equidistant

Try and recreate the above two loci and constructions on paper using a pair of compasses and a pencil and ruler.

ANSWERS

Year 9 Foundation
BEARINGS

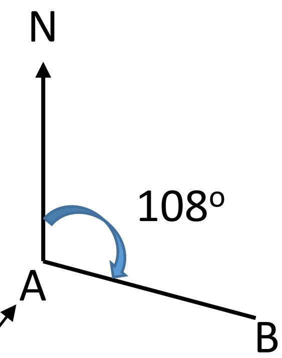
Key Concepts

Bearings are a type of angle that are used in real life directional instructions. They have **three rules** that they must conform to:

1) They must always be **measured from North**.
2) They must always be measured in a **clockwise direction**.
3) They must always have **3 figures** e.g. 72° is written as 072°

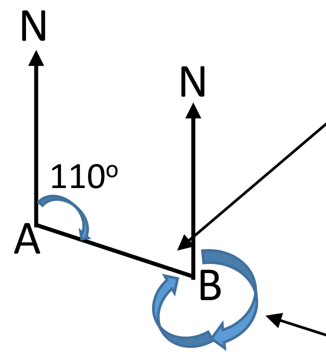
Examples

The bearing of B **from** A is 108°



Where we start measuring from using our **protractor**

We don't always need a protractor to find bearings, we can use our angle facts knowledge.



Because we know co-interior angles sum to 180°, this angle must be 70°.

The angle we are finding is the clockwise angle from B. We know angles around a point sum to 360°.

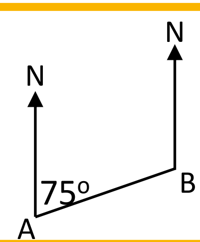
The bearing of A **from** B is 290°

hegartymaths

492-496

Key Words

Bearing
Clockwise
North
Angle
Protractor



The bearing of B **from** A is 075°.
Calculate the bearing of A **from** B.

ANSWERS 255°

Year 9 Foundation
PLANS AND ELEVATIONS

Key Concepts

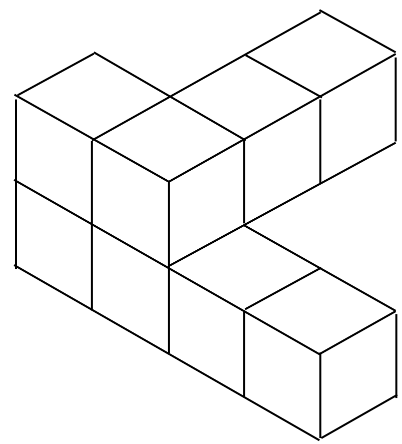
A 3 dimensional shape can be mathematically drawn from **three view points**:

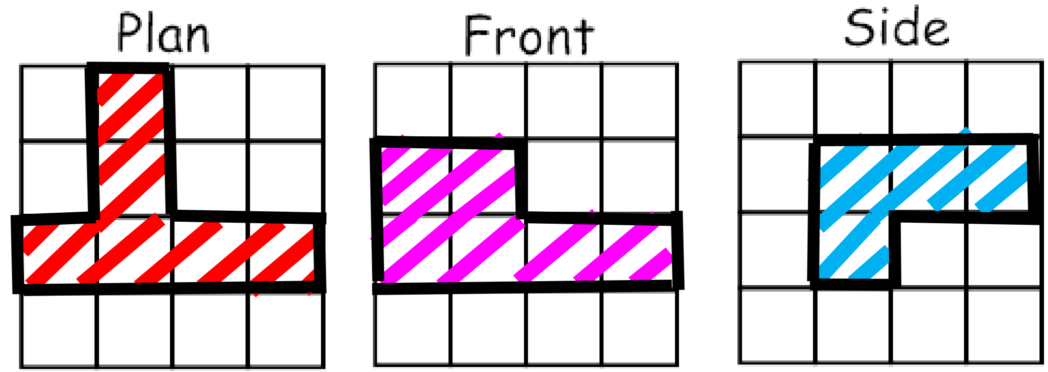
Side view
Front view
Plan view – from above

They are drawn as 2 dimensional representations

Examples

Draw this 3D shape from the side view, the front view and the plan view.

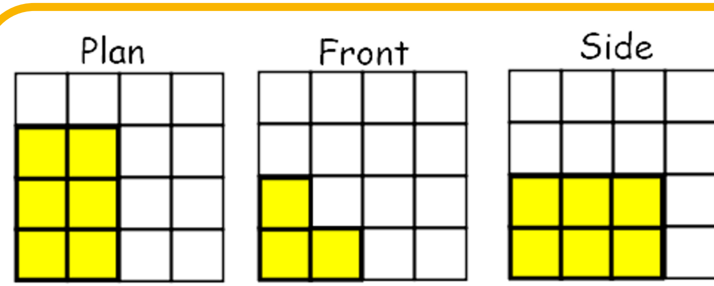




hegartymaths

Key Words

Elevation
Plan
Side
Front



Sketch the 3D shape that has these three views.

ANSWERS

Year 9 Foundation
PYTHAGORAS AND TRIGONOMETRY

Key Concepts

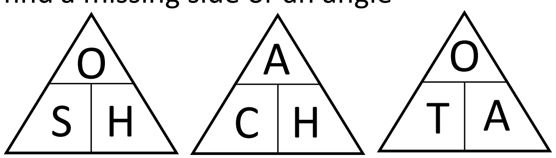
Pythagoras' theorem and basic trigonometry both work with **right angled triangles.**

Pythagoras' Theorem – used to find a missing length when two sides are known

$$a^2 + b^2 = c^2$$

c is always the hypotenuse (the longest side)

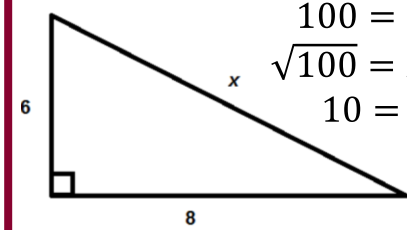
Basic trigonometry SOHCAHTOA – used to find a missing side or an angle



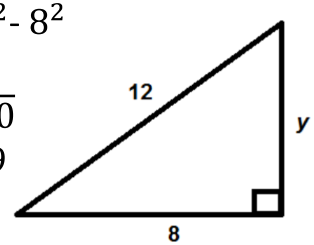
When finding the missing angle we must press **SHIFT** on our calculators first.

Examples

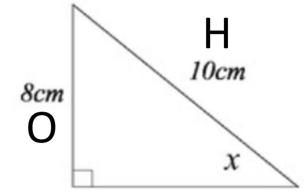
Pythagoras' Theorem

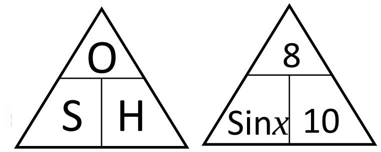


$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = x^2$$
$$100 = x^2$$
$$\sqrt{100} = x$$
$$10 = x$$

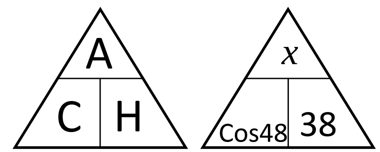


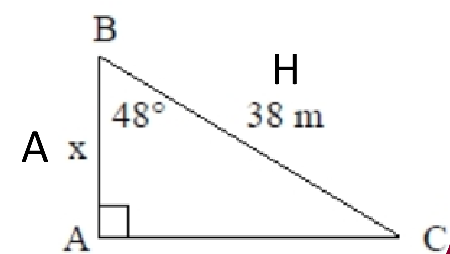
$$a^2 + b^2 = c^2$$
$$a^2 + 8^2 = 12^2$$
$$a^2 = 12^2 - 8^2$$
$$a^2 = 80$$
$$a = \sqrt{80}$$
$$a = 8.9$$





$$\sin x = \frac{8}{10}$$
$$x = \sin^{-1}\left(\frac{8}{10}\right)$$
$$x = 53.1^o$$





$$\cos 48 = \frac{x}{38}$$
$$38 \times \cos 48 = x$$
$$x = 25.4m$$

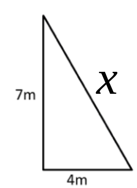
hegartymaths

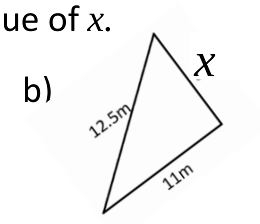
498-499, 509-515

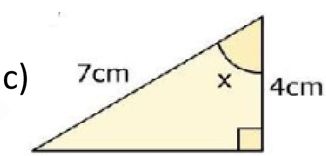
Key Words

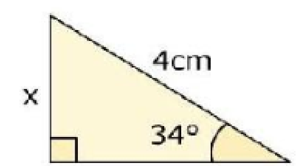
Right angled triangle
Hypotenuse
Opposite
Adjacent
Sine
Cosine
Tangent

Find the value of x.

a) 

b) 

c) 

d) 

ANSWERS: a) 8.06m b) 5.94m c) 55.15° d) 2.34cm

Half Term 4

Year 9 Foundation

AVERAGES FROM A LIST AND REVERSE MEAN

Key Concepts

There are three types of **average** that we use to analyse and compare data. We can calculate averages from a **discrete** data set.

Mode The most common value that appears in the list.

Median Once ordered, the middle value.

Mean $\frac{Total\ of\ all\ data}{Number\ of\ pieces\ of\ data}$

The **range** is used to analyse the **spread** of a data set or how **consistent** the data is.

Range
$largest\ data\ value - smallest\ data\ value$

Examples

Here is a discrete data set, calculate the mean, mode, median and range for this data.

2 5 3 9 7 7

Mode: 7

Median: 2 3 5 7 7 9 $\frac{5+7}{2} = 6$

Mean: $\frac{2+3+5+7+7+9}{6} = 5.5$

Range: 9 – 2 = 7

Reverse mean

A hockey team scored the following number of goals in 6 games:

2 3 4 1 0 1

The mean of the goals scored in seven games was 2. How many goals were scored in the seventh game?

$$\frac{2+3+4+1+0+1+x}{7} = 2 \longrightarrow \frac{11+x}{7} = 2 \longrightarrow x = 3$$

hegartymaths

404-410

Key Words

Discrete
Data
Mean
Mode
Median
Range
Spread

1) Calculate the mean, mode, median and range for the following list of data: 5 8 4 2 8 6

2) The points scored in a test by 5 students are 32, 38, 21, 25, 29. Another students test score is included. If the mean of these 6 scores is now 27, what did the 6th student score?

ANSWERS: 1) mode = 8, median = 5.5, mean = 5.5, range = 6 2) 17

Year 9 Foundation
AVERAGES FROM A TABLE

Key Concepts

Modal class (mode)
Group with the highest frequency.

Median group
The median lies in the group which holds the $\frac{total\ frequency+1}{2}$ position.
Once identified, use the cumulative frequency to identify which group the median belongs from the table.

Estimate the mean
For grouped data, the mean can only be an estimate as we do not know the exact values in each group. To estimate, we use the midpoints of each group and to calculate the mean we find $\frac{total\ fx}{total\ f}$.

Examples

Length (L cm)	Frequency (f)	Midpoint (x)	fx
$0 < L \leq 10$	10	5	$10 \times 5 = 50$
$10 < L \leq 20$	15	15	$15 \times 15 = 225$
$20 < L \leq 30$	23	25	$23 \times 25 = 575$
$30 < L \leq 40$	7	35	$7 \times 35 = 245$
Total	55		1095

a) Estimate the mean of this data.
step 1: calculate the total frequency
step 2: find the midpoint of each group
step 3: calculate $f \times x$
step 4: calculate the mean shown below

$$\frac{Total\ fx}{Total\ f} = \frac{1095}{55} = 19.9\text{cm}$$

b) Identify the modal class from this data set. ***" the group that has the highest frequency "***
Modal class is $20 < x \leq 30$

c) Identify the group in which the median would lie. Median $= \frac{Total\ frequency+1}{2} = \frac{56}{2} = 28th\ value$
" add the frequency column until you reach the 28th value " Median is the in group $20 < x \leq 30$

hegartymaths
414-418

Key Words
Midpoint
Mean
Median
Modal

Cost (£C)	Frequency	Midpoint	
$0 < C \leq 4$	2		
$4 < C \leq 8$	3		
$8 < C \leq 12$	5		
$12 < C \leq 16$	12		
$16 < C \leq 20$	3		

From the data:
a) Identify the modal class.
b) Identify the group which holds the median.
c) Estimate the mean.

ANSWERS: a) $12 < C \leq 16$ b) $\frac{25+1}{2} = 13^{th}$ value is in the group $12 < C \leq 16$ c) $\frac{294}{25} = £11.76$

Year 9 Foundation
TYPES OF DATA AND GRAPHS

Key Concepts

Qualitative data: data collected that is described in words **not** numbers.
e.g. race, hair colour, ethnicity.

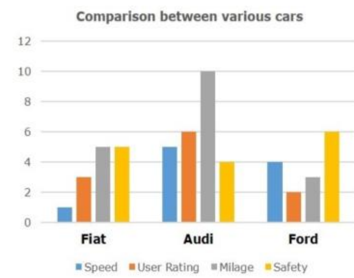
Quantitative data: this is the collection of numerical data that is either <u>discrete</u> or <u>continuous</u>.

Discrete data: numerical data that is categorised into a finite number of classifications.
e.g. number of siblings in a family, shoe size, .

Continuous data: numerical data that can take any value. This data is usually measured on a large number scale.
e.g. height, weight, time, capacity.

Examples

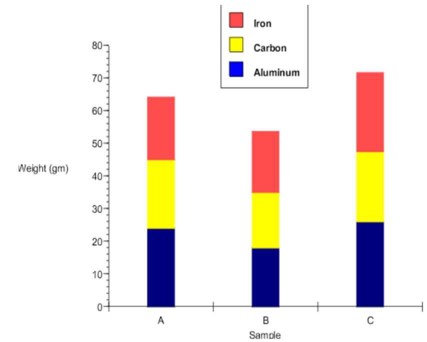
Comparative bar charts



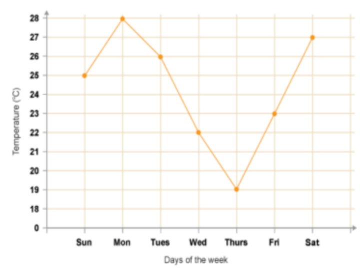
Tally charts

Colour	Tally	Frequency
Red	卌 卌 III	13
Blue	卌 IIII	9
White	卌 卌 卌 卌 IIII	24
Black	卌 卌 II	12
Other	卌 IIII	9

Composite bar charts

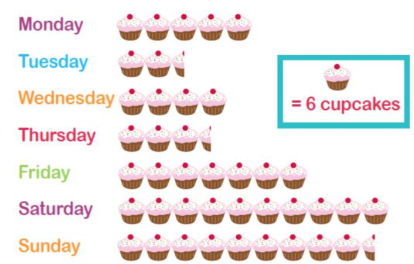


Line graphs



Pie charts

Pictograms



hegartymaths

425,426,427,
430-433,442

Key Words

Data
Discrete
Continuous
Qualitative
Quantitative
Graph

What types of data is each of the following?

1) Eye colour
2) Time it takes to run 100m
3) Number of goals scored in a match
4) Length of a car (to the nearest cm)
5) Number of pets a person owns

ANSWERS: 1) Qualitative 2) Continuous, quantitative 3) Discrete, quantitative 4) Continuous, quantitative 5) Discrete, quantitative

Year 9 Foundation
BAR CHARTS AND PICTOGRAMS

Key Concepts

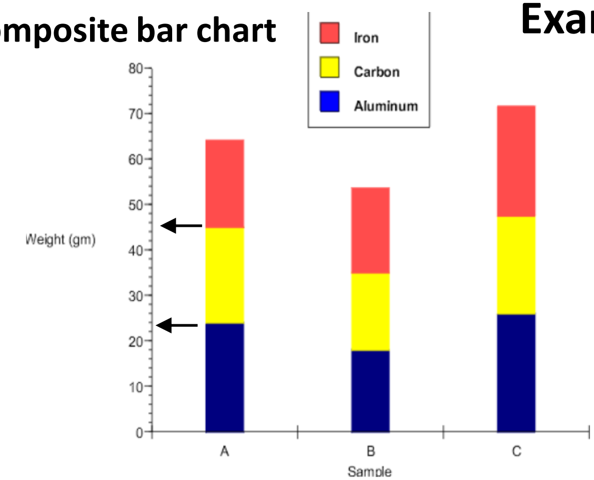
Bar charts are a visual representation of **categorical** data.

Composite bar charts are bar charts that display multiple data points stacked on top of one another.

Pictograms uses an image relating to a physical object to represent an amount. A **key** must be included to show the value of each picture.

Examples

Composite bar chart



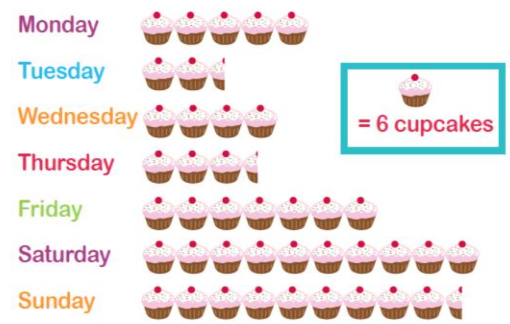
1) How much aluminium is in sample A? 24g
2) Hoe much carbon is in sample A?

46 – 24 = 22g

Highest value for carbon in sample A.

Lowest value for carbon in sample A.

Pictogram



1) How many cupcakes were sold on Monday?

$5 \times 6 = 30$ cupcakes

2) What does half a cupcake represent on the pictogram?

$6 \div 2 = 3$ cupcakes

3) How many cupcakes were sold on Thursday?

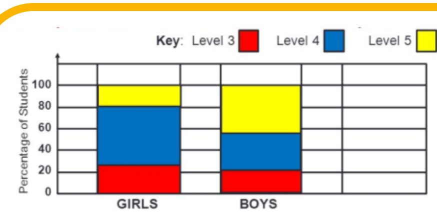
$3.5 \times 6 = 21$ cupcakes

hegartymaths

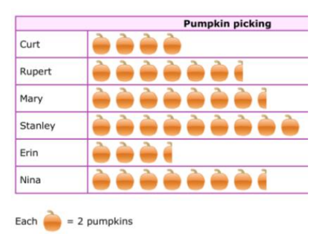
425-426

Key Words

Bar chart
Composite
Pictogram
Key
Categorical
Data set



1a) What percentage of boys are level 3?
b) What percentage of girls are level 4?



2a) How many pumpkins were picked by Stanley?
b) What does half a pumpkin represent?
c) How many pumpkins were picked by Erin?

ANSWERS: 1a) 20% b) approximately 55% 2a) 18 b) 1 pumpkin c) 7

Year 9 Foundation

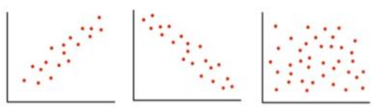
PIE CHARTS AND SCATTER-GRAPHS

Key Concepts

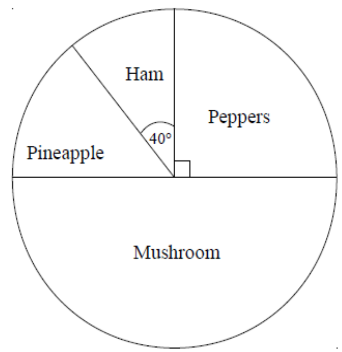
Pie charts use angles to represent, proportionally, the quantity of each group involved.

Pie charts can only be compared to one another when the total frequency or populations are given.

Scatter-graphs show the relationship between two variables. This relationship is called the **correlation**.

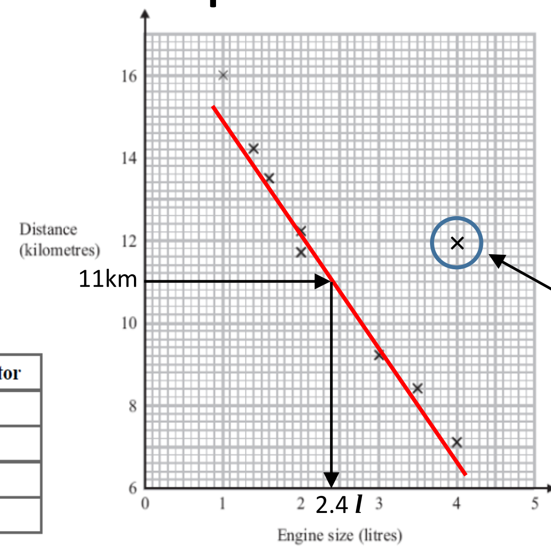


Examples



Topping	Frequency	Angle of Sector
Peppers	18	90°
Mushroom	36	180°
Pineapple	10	50°
Ham	8	40°
	Total=72	360°

$360^\circ \div 72$ → $\times 5$



A scatter-graph is drawn to show the relationship between the engine size of a car and how far it can travel.

It shows negative correlation.

This is an **outlier**. It does not match the trend.

We draw a **line of best fit** through the data points to help estimate readings, based on the data sample. For example, estimating the engine size of a car that can travel 11km would be 2.4 litres.

hegartymaths

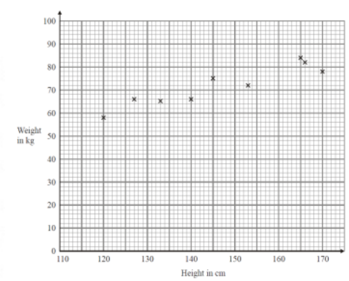
427-429, 453,454

Key Words

Pie chart
Scatter-graph
Correlation
Outlier
Variable

1) Calculate the angle for each category:

Region	Frequency
Southern England	9
London	23
Midlands	16
Northern England	12
Total	60



2a) What type of correlation is shown?
b) Using a line of best fit estimate the weight when the height is 135cm.

ANSWERS: 1) 360°÷60=6°, Angles = 54°, 138°, 96°, 72° 2) a) positive b) 64kg-66kg

Half Term 5

Year 9 Foundation
RELATIVE FREQUENCY

Key Concepts

Experimental probability differs to theoretical probability in that it is based upon the **outcomes from experiments**. It may not reflect the outcomes we expect.

Experimental probability is also known as the **relative frequency** of an event occurring.

Estimating the number of times an event will occur:

Probability × no. of trials

Examples

Colour	red	blue	white	black
Prob	x	0.2	0.3	x

A spinner is spun, it has four colours on it.
The relative frequencies of each colour are recorded.
The relative frequency of red and black are the same.

a) What is the relative frequency of red?

$$1-(0.2+0.3)=0.5$$

$$x=\frac{0.5}{2}=0.25$$

b) If the spinner is spun 300 times, how many times do you expect it to land on white?

$$0.3\times 300=90$$

hegartymaths

355-357

Key Words

Experimental
Relative frequency
Fraction
Decimal
Probability
Estimate

Number	1	2	3	4
Prob	x	0.46	0.28	x

A spinner is spun which has 1,2,3,4 on it. The probability that a 1 and a 4 are spun are equal.

a) What is the probability that a 4 is landed on?

b) If the spinner is spun 500 times how many times do we expect it to land on a 2?

ANSWERS: a) 0.13 b) 230

Year 9 Foundation
THEORETICAL PROBABILITY

Key Concepts

Probabilities can be described using **words** and **numerically**.

We can use **fractions, decimals or percentages** to represent a probability.

Theoretical probability is what should happen if all variables were fair.

All probabilities must **add to 1**.

The probability of something **NOT** happening equals:

$$1 - (probability\ of\ it\ happening)$$

Examples

Probability scale:

Impossible		Even chance		Certain
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{4}$
0	0.25	0.5	0.75	1
0%	25%	50%	75%	100%

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	3	5	2

1) What is the probability that a blue counter is chosen? $\frac{3}{19} = \frac{number\ of\ blue}{total\ number\ of\ counters}$

2) What is the probability that red is **not** chosen? $\frac{10}{19} = \frac{number\ of\ all\ other\ colours}{total\ number\ of\ counters}$

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	$3x$	x-5	$2x$

A counter is chosen at random, the probability it is red is $\frac{9}{100}$. Work out the probability is black.

$$9 + 3x + x - 5 + 2x = 100$$
$$6x + 4 = 100$$
$$x = 16$$

Number of black counters = 16 – 5

= 11

Probability of choosing black = $\frac{11}{100}$

hegartymaths

349-353

Key Words

Theoretical
Probability
Fraction
Decimal
Percentage
Certain
Impossible
Even chance

	1	2	3
Prob	5	4	9

1a) Calculate the probability of choosing a 2.
b) Calculate the probability of not choosing a 3.

	1	2	3
Prob	0.37	$2x$	x

2) Calculate the probability of choosing a 2 or a 3.

ANSWERS: 1a) $\frac{4}{18}$ b) $\frac{6}{18}$ 2) P(2) = 0.42 P(3) = 0.21

Year 9 Foundation

LISTING OUTCOMES AND SAMPLE SPACE

Key Concepts

When there are a number of different possible outcomes in a situation we need a **logical** and **systematic** way in which to view them all.

We can be asked to **list** all possible outcomes e.g. choices from a menu, order in which people finish a race.

We can also use a **sample space diagram**. This records the possible outcomes of two different events happening.

Examples

Starter	Main
Fishcake Melon	Lasagne Beef Salmon

List all of the combinations possible when one starter and one main are chosen.

F, L	M, L
F, B	M, B
F, S	M, S

Note: You can write the initials of each option in a test. You do not need to write out the full word.

Two dice are thrown and the possible outcomes are shown in the sample space diagram below:

	1	**2**	**3**	**4**	**5**	**6**
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

1) What is the probability that 2 numbers which are the same are rolled?

$$\frac{6}{36} = \frac{\textit{outcomes where numbers are the same}}{\textit{total number of outcomes}}$$

2) What is the probability that two even numbers are rolled?

$$\frac{9}{36} = \frac{\textit{outcomes where numbers are both even}}{\textit{total number of outcomes}}$$

hegartymaths

358-359, 370-371

Key Words

List

Outcome

Sample space

Probability

1) Abe, Ben and Carl have a race. List all of the options for the order that the boys can end the race.

		Spinner		
Coin		Red	Green	Blue
	Heads	H,R	H,G	H,B
	Tails	T,R	T,G	T,B

2a) What is the probability that a head is landed on?
b) What is the probability that a head and a green are landed on?

ANSWERS: 1) ABC, ACB, BAC, BCA, CAB, CBA 2a) $\frac{3}{6}$ b) $\frac{1}{6}$

Year 9 Foundation
VENN DIAGRAMS

Key Concepts

Venn diagrams show all possible relationships between different sets of data.

Probabilities can be derived from Venn diagrams. Specific notation is used for this:

P(A ∩ B) = Probability of A **and** B

P(A ∪ B) = Probability of A **or** B

P(A') = Probability of **not** A

hegartymaths

372-388, 391

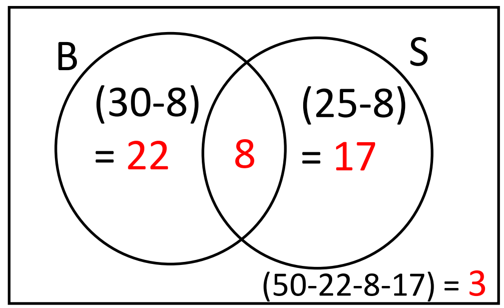
Key Words

Venn diagram
Union
Intersection
Probability
Outcomes

Example

Out of 50 people surveyed:

30 have a brother
25 have a sister
8 have both a brother and sister



a) Complete the Venn diagram

b) Calculate:

i) P(A ∩ B) $= \frac{8}{50}$ ii) P(A ∪ B) $= \frac{47}{50}$ iii) P(B') $= \frac{20}{50}$

iv) The probability that a person with a sister, does not have a brother.

$= \frac{8}{25}$

40 students were surveyed:

20 have visited France
15 have visited Spain
10 have visited both France and Spain

a) Complete a Venn diagram to represent this information.

b) Calculate:

i) P(F ∩ S) ii) P(F ∪ S) iii) P(S')

iv) The probability someone who has visited France, has not gone to Spain.

ANSWERS: b) i) 10/40 ii) 25/40 iii) 25/40 iv) 10/40 = 1/4

Year 9 Foundation

PROBABILITY TREE DIAGRAMS

Key Concepts

Independent events are events which do not affect one another.

Dependent events affect one another's probabilities. This is also known as **conditional probability**.

We **multiply** two probabilities when one event follows another.

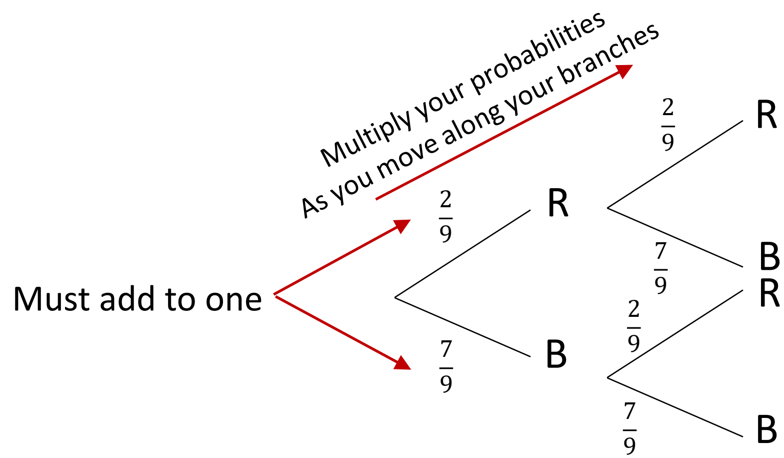
Examples

There are red and blue counters in a bag.

The probability that a red counter is chosen is $\frac{2}{9}$.

A counter is chosen and **replaced**, then a second counter is chosen.

Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



Prob of two reds:

$$\frac{2}{9} \times \frac{2}{9} = \frac{4}{81}$$

Prob of two blues :

$$\frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$$

Prob of same colours:

$$\frac{4}{81} + \frac{49}{81} = \frac{53}{81}$$

hegartymaths

361-362, 364, 368-369

Key Words

Independent
Dependant
Conditional
Probability
Fraction
Multiply

There are blue and green pens in a drawer.
There are 4 blues and 7 greens.
A pen is chosen and then **replaced**, then a second pen is chosen.
Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

ANSWERS: 56/121

Half Term 6

Year 9 Foundation

TYPES OF ANGLE AND ANGLES IN POLYGONS

Key Concepts

Regular polygons have equal lengths of sides and equal angles.

Angles in polygons

Sum of interior angles

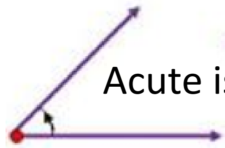
$= (number\ of\ sides - 2) \times 180$

Exterior angles of **regular** polygons = $\frac{360}{number\ of\ sides}$

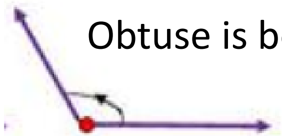
Types of angle

There are four types which need to be identified – acute, obtuse, reflex and right angled.

Examples



Acute is less than 90°

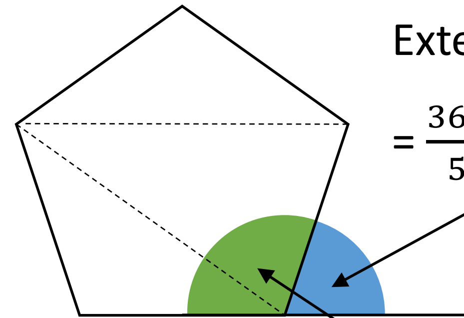


Obtuse is between 90° and 180°

Right angled is 90°

Reflex is between 180° and 360°

Regular Pentagon



Exterior angles

$= \frac{360}{5} = 72^o$

Sum of interior angles

$= (5 - 2) \times 180$

$= 540^o$

Interior angle = $\frac{540}{5} = 108^o$

hegartymaths

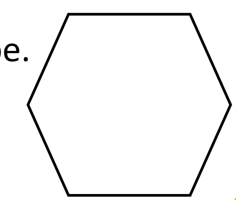
455, 456, 560-564

Key Words

Polygon
Interior angle
Exterior angle
Acute
Obtuse
Right angle
Reflex

Questions

1) Calculate the sum of the interior angles for this regular shape.
2) Calculate the exterior angle for this regular shape.
3) Calculate the size of one interior angle in this regular shape.



ANSWERS: 1) 720° 2) 60° 3) 120°

Year 9 Foundation

ANGLE FACTS INCLUDING ON PARALLEL LINES

Key Concepts

Angles in a **triangle equal 180°**.

Angles in a **quadrilateral equal 360°**.

Vertically opposite angles are equal in size.

Angles on a **straight line equal 180°**.

Base angles in an isosceles triangle are equal.

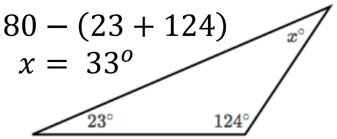
Alternate angles are equal in size.

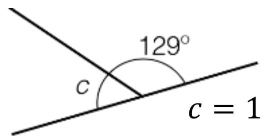
Corresponding angles are equal in size.

Allied/co-interior angles are equal 180°.

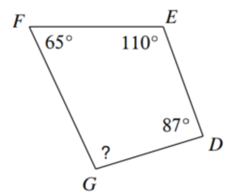
Examples

$x = 180 - (23 + 124)$
$x = 33^o$

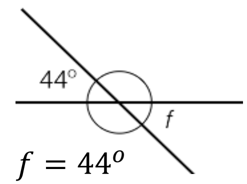




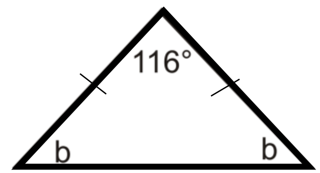
$c = 180 - 129$
$x = 51^o$



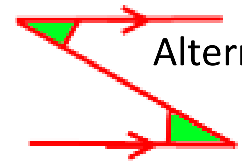
$? = 360 - (65 + 110 + 87)$
$? = 98^o$



$f = 44^o$

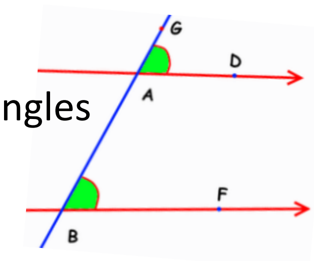


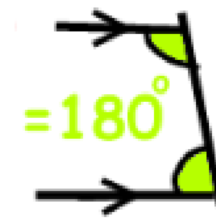
$b = (180 - 116) \div 2$
$b = 32^o$



Alternate angles are equal

Corresponding angles are equal





Allied/co-interior angles equal 180°

hegartymaths

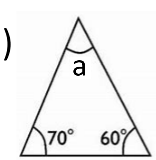
477-480, 481-483

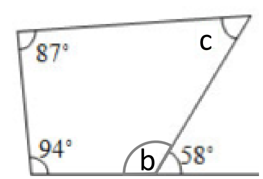
Key Words

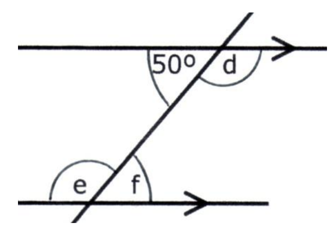
Angle
Vertically opposite
Straight line
Alternate
Corresponding
Allied
Co-interior

Questions

Calculate the missing angle:

a) 

b) 

c) 

ANSWERS: 1) a=50° 2) b=122° c=57° 3) d=130° e=130° f=50°