# Year 8 Maths Knowledge Organiser (F)

# Half term 1

# FOUR OPERATIONS WITH INTEGERS & DECIMALS

## Key Words

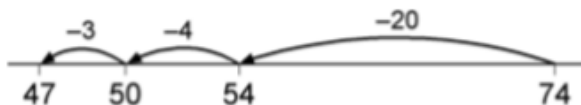**Place Value:** The value a digit takes when placed in a particular position of a number.

| | |
|---|---|
| ➕ Add Sum Total All together Plus In all | ✖ Multiply Product Times Twice Total Multiplied by |
| ➖ Subtract Remain Difference Less than Fewer How many more Minus | ➗ Divide Quotient Goes into Split Equally Each |

## Examples

$48 + 36 = 84$

+30   +2   +4

48        78  80   84

$74 - 27 = 47$ worked by counting back:

−3   −4      −20

47   50   54            74

$$\begin{array}{r} 258 \\ + \ 87 \\ \hline 345 \\ 11 \end{array}$$

$$\begin{array}{c} {\color{green}3} \\ 4\,{\color{green}1}5\,- \\ 2\ \ 8 \\ \hline {\color{red}1}\ {\color{red}7} \end{array}$$

$$\begin{array}{r} 38 \\ \times \ 7 \\ \hline 56 \\ 210 \\ \hline 266 \end{array}$$

$56 \times 27$

| × | 20 | 7 | |
|---|---|---|---|
| 50 | 1000 | 350 | 1350 |
| 6 | 120 | 42 | 162 |
| | | | 1512 |
| | | | 1 |

$$3\overline{)2\ 9^2 1} = 97$$

## Tip

Multiplication and addition are associative, so you can work them out in any order.
So 3 × 4 is the same as 4 × 3.
4 + 3 is the same as 3 + 4

## Questions

1) a) $49 + 37$   b) $125 + 69$   c) $5.6 + 24.8$
2) a) $64 - 28$   b) $134 - 57$   c) $16.2 - 9.5$
3) a) $7 \times 146$   b) $34 \times 67$   c) $2.9 \times 7.2$   4) a) $294 \div 7$   b) $192 \div 6$

ANSWERS : 1) a) 86  b) 194  c) 30.4    2) a) 36  b) 77  c) 6.7
3) a) 1022  b) 2278  c) 20.88    4) a) 42  b) 32

# INTEGERS, ROUNDING AND PLACE VALUE

## Key Concepts

Digits are the individual components of a number.

Integers are whole numbers.

Rounding rules:
A value of 5 to 9 rounds the number up.
A value of 0 to 4 keeps the number the same.

## Examples

**Order** the following numbers starting with the smallest:

1)     5,  -3,  4,  7,  -2
       -3,  -2,  4, 5, 7

2)          0.067  0.6      0.56    0.65   0.605
Rewrite   0.067, 0.600, 0.560, 0.650, 0.605
          0.067  0.56    0.6      0.605  0.65

**Round** 3.527 to:

a) 1 decimal place

     3 . 5 2 7 → 3.5

b) 2 decimal places

     3 . 5 2 7 → 3.53

c) 1 significant figure

     3 . 5 2 7 → 4

## Key Words

Integer        Even
Digit          Odd
Decimal place
Significant figures

A) Order the following numbers starting with the smallest:
1)  6, -2, 0, -5, 3     2) 0.72, 0.7, 0.072, 0.07, 0.702

B)     Round the following numbers to the given degree of accuracy
1)     14. 1732     (1 d.p.)    2) 0.0568   (2 d.p.)   3)3418  (1 S.F)

# CALCULATIONS, CHECKING AND ROUNDING

## Key Concepts

A value of 5 to 9 rounds the number up.

A value of 0 to 4 keeps the number the same.

Estimation is a result of rounding to one significant figure.

## Examples

**Round** 3.527 to:

a) 1 decimal place

$$3 . 5 \,|\, 2 \, 7 \longrightarrow 3.5$$

b) 2 decimal places

$$3 . 5 \, 2 \,|\, 7 \longrightarrow 3.53$$

c) 1 significant figure

$$3 \,|\, 5 \, 2 \, 7 \longrightarrow 4$$

**Estimate** the answer to the following calculation:

$$\frac{46.2 - 9.85}{\sqrt{16.3 + 5.42}}$$

$$\frac{50 - 10}{\sqrt{20 + 5}}$$

$$\frac{40}{5} = 8$$

## Key Words

Integers
Operation
Negative
Significant figures
Estimate

A)     Round the following numbers to the given degree of accuracy
1)     14.1732     (1 d.p.)     2) 0.0568   (2 d.p.)     3)3418  (1 S.F)
B)     Estimate:
1)     $\sqrt{4.09 \times 8.96}$

2)     $25.76 - \sqrt{4.09 \times 8.96}$

3)     $\sqrt[3]{26.64} + \sqrt{80.7}$

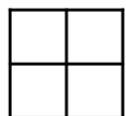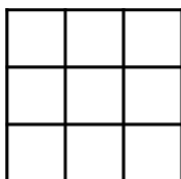4)     $\frac{\sqrt{6.91 \times 9.23}}{3.95^2 \div 2.02^3}$

ANSWERS  A 1) 14.2  2) 0.06  3) 3000  B 1) 6  2) 24  3) 12  4) 4

# POWERS AND ROOTS

## Key Concept

**Square numbers**

$1^2$
$1 \times 1 = 1$

$2^2$
$2 \times 2 = 4$
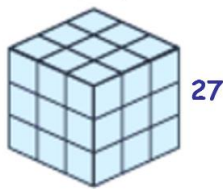
$3^2$
$3 \times 3 = 9$

**Cube numbers**

1
$1^3$
$1 \times 1 \times 1$

8
$2^3$
$2 \times 2 \times 2$

27
$3^3$
$3 \times 3 \times 3$

## Key Words

**Square:** A square number is the result of multiplying a number by itself.

**Cube:** A cube number is the result of multiplying a number by itself twice.

**Root:** A root is the reverse of a power.

**Prime number:** A prime is a number that has only two factors which are 1 and itself.

**Reciprocal:** This is found by doing 1 divided by the number.

**Factor:** A number that fits into another number exactly.

## Tip

A number with an odd amount of factors must be a square number.

## Examples

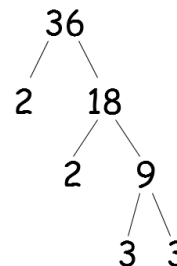What is $2^4$ ?                    $2 \times 2 \times 2 \times 2 = 16$

What is $\sqrt{64}$ ?              $8^2 = 64$, so $\sqrt{64} = \pm 8$

What is the reciprocal of 5?       $\frac{1}{5}$

Write 36 as a product of prime factors

36
2   18
2   9
3   3

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

Product means 'multiply'

## Questions

1)  a) $2^5$    b) $3^3$    c) $1^{17}$    d) $\sqrt{81}$    e) $\sqrt{16}$    f) $\sqrt[3]{64}$

2)  Find the reciprocal of:    a)  4    b) $\frac{1}{3}$    c) 0.25

3)  Write 72 as a product of primes.

# INDICES AND ROOTS

## Key Concepts

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m}$$

## Examples

**Simplify** each of the following:

1) $a^6 \times a^4 = a^{6+4}$
$$= a^{10}$$

2) $3^6 \times 3^5 = 3^{6+5}$
$$= 3^{11}$$

3) $a^6 \div a^4 = a^{6-4}$
$$= a^2$$

4) $9^6 \div 9^3 = 9^{6-3}$
$$= 9^3$$

5) $(a^6)^4 = a^{6\times4}$
$$= a^{24}$$

6) $(3a^4)^3 = 3^3 a^{4\times3}$
$$= 27a^{12}$$

7) $a^{-1} = \frac{1}{a^1}$

8) $a^{-2} = \frac{1}{a^2}$

9) $a^{\frac{1}{2}} = \sqrt[2]{a^1} = \sqrt{a}$

**102 – 110**

## Key Words
Powers
Roots
Indices
Reciprocal

Write as a single power:   1) $a^3 \times a^2$   2) $b^4 \times b$   3) $d^{-5} \times d^{-1}$   4) $m^6 \div m^2$

5) $n^4 \div n^4$   6) $\frac{8^4 \times 8^5}{8^6}$   7) $\frac{4^9 \times 4}{4^3}$

Evaluate :   1) $(3^2)^5$   2) $2^{-2}$   3) $81^{\frac{1}{2}}$   4) $27^{\frac{1}{3}}$

ANSWERS: 1) $a^5$   2) $b^5$   3) $d^{-6}$   4) $m^4$   5) 1   6) $8^3$   7) $4^7$
1) $3^{10}$   2) $\frac{1}{4}$   3) 9   4) 3

# FACTORS, MULTIPLES AND PRIMES

## Key Concepts

**Prime factor decomposition**
Breaking down a number into its prime factors
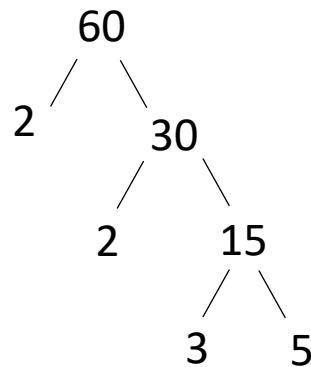
**Highest common factor**
Finding the largest number which divides into all numbers given

**Lowest common multiple**
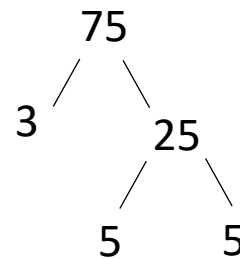Finding the smallest number which both numbers divide into

## Examples

Find the **highest common factor** and **lowest common multiple** of 60 and 75:
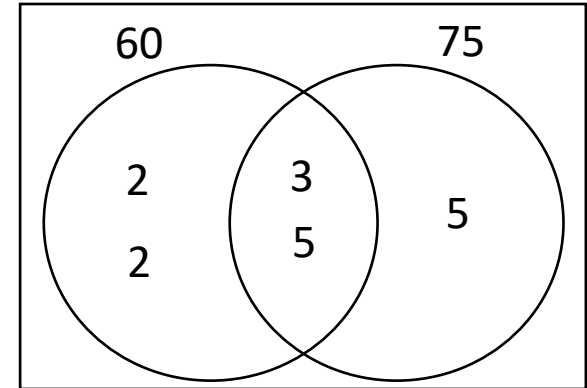
60
2   30
    2   15
        3   5

$2 \times 2 \times 3 \times 5$
$2^2 \times 3 \times 5$

75
3   25
    5   5

$3 \times 5 \times 5$
$3 \times 5^2$

Venn diagram: left circle (60) contains 2, 2; intersection contains 3, 5; right circle (75) contains 5

$HCF$ − Mulitiply all numbers in the intersection
$$= 3 \times 5 = 15$$

$LCM$ − Multiply all numbers in the Venn diagram
$$= 2 \times 2 \times 3 \times 5 \times 5 = 300$$

## Key Words
Factor
Multiple
Prime
Highest Common Factor
Lowest Common
Multiple

## Questions
1) Write 80 as a product of its prime factors
2) Write 48 as a product of its prime factors
3) Find the LCM and HCF of 80 and 48

ANSWERS: 1) $2^4 \times 5$  2) $2^4 \times 3$  3) LCM = 240 and HCF = 16

# Half term 2

# SIMPLIFYING & MANIPULATING ALGEBRA

## Key Concept

**Formula**

$$v = u + at$$

**Expression**

$$f^2 + f^2 + f^2$$

**Equation**

$$34 = 12 + 6t$$

**Identity**

$$c \times c = c^2$$

## Key Words

**Formula:** A rule written using symbols that describe a relationship between different quantities.
**Expression:** Shows a mathematical relationship whereby there is no solution.
**Equation:** A mathematical statement that shows that two expressions are equal.
**Identity:** A relation which is true. No matter what values are chosen.

**Tip**
When expanding brackets be careful with negatives.

## Examples

Simplify:



$$4a + 3b - a + 2b$$
$$= 3a + 5b$$

Expand and simplify:

$$9a - 2(3a - 4)$$
$$9a - 6a + 8$$
$$3a + 8$$

Factorise:

$$9x^2 + 6x$$

Factorising is the opposite of expanding brackets

$3x$ is common to both terms

$$3x(3x + 2)$$

Expand and simplify:

$$2(4a + 2b) - 2(a + 3b)$$



$$8a + 4b - 2a - 6b$$
$$6a - 2b$$

### Questions

1) $5x + 3y - 2x + 4y$     2) $2p - 6q + 2q + 4p$     3) $12b - 3(2b + 5)$
4) Factorise  a)  $4x + 10$     b) $8a^2 - 10a$

ANSWERS:  1) $3x + 7y$   2) $6p - 4q$   3) $6b - 15$   4) a) $2(2x + 5)$   b) $2a(4a - 5)$

# EXPRESSIONS/EQUATIONS/IDENTITIES AND SUBSTITUTION

## Key Concepts

A **formula** involves two or more letters, where one letter equals an **expression** of other letters.

An **expression** is a sentence in algebra that does NOT have an equals sign.

An **identity** is where one side is the equivalent to the other side.

When **substituting** a number into an expression, replace the letter with the given value.

## Examples

1) $5(y + 6) \equiv 5y + 30$  is an identity as when the brackets are expanded we get the answer on the right hand side

2) $5m – 7$  is an expression since there is no equals sign

3) $3x – 6 = 12$ is an equation as it can be solved to give a solution

4) $C = \frac{5(F - 32)}{9}$  is a formula (involves more than one letter and includes an equal sign)

5) Find the value of $3x + 2$ when $x = 5$

$(3 \times 5) + 2 = 17$

6) Where A = b² + c, find A when b = 2 and c = 3

$A = 2^2 + 3$

$A = 4 + 3$

$A = 7$

## Key Words
Substitute
Equation
Formula
Identity
Expression

## Questions

1) **Identify the equation, expression, identity, formula from the list** (a) $v = u + at$      (b) $u^2 - 2as$

     (c) $4x(x - 2) = x^2 - 8x$      (d) $5b – 2 = 13$

2) Find the value of $5x – 7$ when $x = 3$

3) Where A = d² + e, find A when d = 5 and e = 2

ANSWERS: 1) (a) formula   (b) expression   (c) identity   (d) equation    2) 8   3) A = 27

# EXPANDING

## Examples

## Expand and simplify:

1)     $4(m + 5) + 3$

   $= 4m + 20 + 3$
   $= 4m + 23$

2)   $(p + 2)(2p - 1)$

   $= p^2 + 4p - p - 2$
   $= p^2 + 3p - 2$

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**160-166, 168, 169**

**Key Words**
Expand
Bracket

A) Expand:
1)   $5(m - 2) + 6$   2) $(5g - 4)(2g + 1)$

# REARRANGE AND SOLVE EQUATIONS

## Key Concepts

**Solving equations:**
Working with inverse operations to find the value of a variable.

**Rearranging an equation:**
Working with inverse operations to isolate a highlighted variable.

In solving and rearranging we **undo the operations** starting from the last one.

For each step in solving an equation we must do the **inverse** operation

Solve:

$5(x - 3) = 20$

Expand

$5x - 15 = 20$

+15         +15

$5x = 35$

÷5         ÷5

$x = 7$

Solve:

$12 = 3x - 18$

+18         +18

$30 = 3x$

÷3         ÷3

$x = 10$

Solve:

$7p - 5 = 3p + 3$

-3p         -3p

$4p - 5 = 3$

+5         +5

$4p = 8$

÷ 2         ÷ 2

$p = 2$

## Examples

**Rearrange** to make $r$ the subject of the formulae :

$$Q = \frac{2r - 7}{3}$$

×3         × 3

$$3Q = 2r - 7$$

+7         +7

$$3Q + 7 = 2r$$

÷ 2         ÷ 2

$$\frac{3Q + 7}{2} = r$$

### Key Words
Solve
Rearrange
Term
Inverse operation

1) Solve $7(x + 2) = 35$

2) Solve $4x - 12 = 28$

3) Solve $4x - 12 = 2x + 20$

4) Rearrange to make $x$ the subject:

$$y = \frac{3x + 4}{2}$$

# SEQUENCES

## Key Concept

**Types of Sequence**

Sequence as pictures:
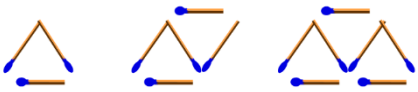


Linear sequence:

4, 7, 10, 13, 16, …

+3  +3  +3  +3

Fibonacci sequence:
(add the previous two terms)

1, 1, 2, 3, 5, 8, …

## Key Words

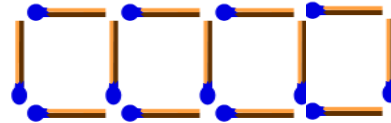**Sequence:** A list which is in a particular order following a pattern.
**Term:** Each particular part of a sequence.
**Linear sequence:** A sequence which is formed by adding or subtracting the same amount each time.

## Examples
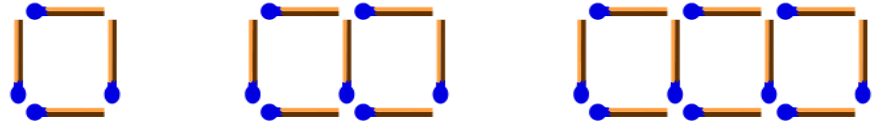


Next pattern is:



Sequence = 4, 7, 10, 13, ….
Term to term rule = + 3
Nth term        4, 7, 10, 13, 16, …   **= 3n + 1**

+1        +3   +3   +3   +3

## Tip

If a sequence is decreasing, the 'n' term will be negative.
Eg, 15, 11, 7, 3, …
Nth term = -4n + 19

## Questions

1) Find the next two terms and the term to term rule
a) 9, 13, 17, 21, …   b) 7, 12, 17, 22, …   c) 9, 7, 5, 3, …   d) 3, 4, 7, 11, 18
2) Find the nth term   a) 7, 9, 11, 13, …        b) 8, 13, 18, 23, …
c) 15, 12, 9, 6, …        d) 1, -3, -7, -11, …

ANSWERS: 1) a) 25, 29 Rule = +4  b) 27, 32, Rule = +5  c) 1, -1, Rule = - 2  d) 29, 47, Rule = add previous 2 numbers    2) a) 2n + 5   b) 5n + 3   c) -3n + 18   d) –4n + 5

# STRAIGHT LINE GRAPHS AND EQUATION OF A LINE

## Key Concepts

**Coordinates** in 2D are written as follows:

$x$ is the value that is to the left/right $(x, y)$ $y$ is the value that is to up/down

**Straight line graphs** always have the equation:

$$y = mx + c$$

$m$ is the **gradient** i.e. the steepness of the graph.
$c$ is the **y intercept** i.e. where the graph cuts the y axis.

## Examples

Plot the graph of $y = x + 1$

| $x$ | 0 | 1 | 2 |
|-----|---|---|---|
| y | 1 | 2 | 3 |
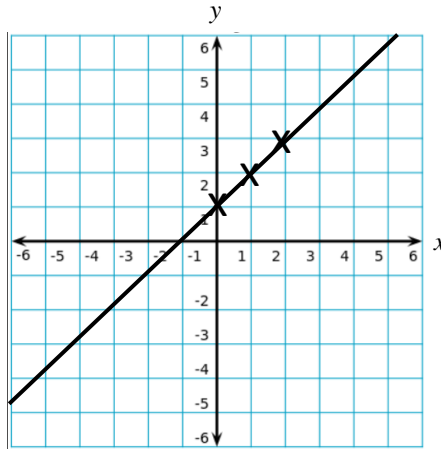


Calculate the equation of this line:



$$y = mx + c$$

$$m = \frac{4}{2}$$

$$= 2$$

$$y = 2x + c$$

Substitute in a coordinate: (2,7)

$$7 = (2 \times 2) + c$$

$$3 = c$$

$$y = 2x + 3$$
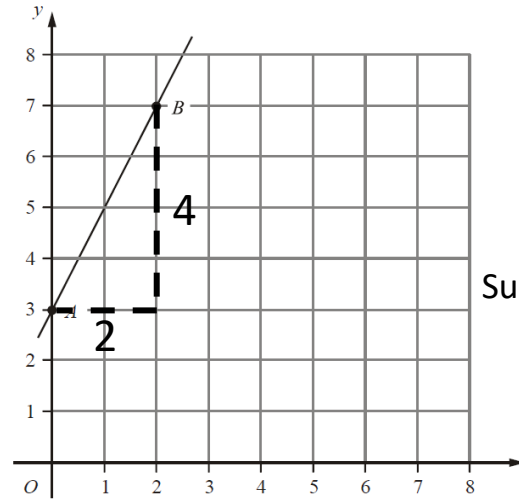
**Key Words**
**Coordinate**
**Gradient**
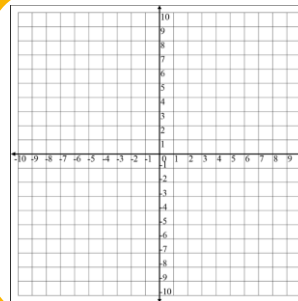
1) Plot the line $y = 3x - 2$

2) Find the equation of the line for the attached graph.



ANSWERS: 2) $y = 2x + 4$
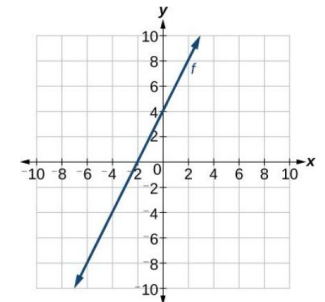
# APPLIED GRAPHS

## Key Concept
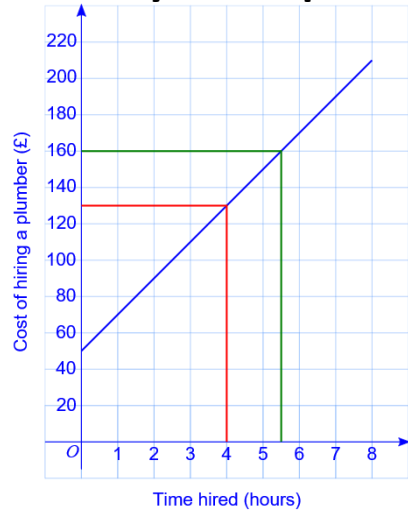


**Gradient** – The extra cost incurred for every extra hour.
**y-intercept** – The minimum payment to the plumber.

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**207-209, 218, 219, 712, 713**

## Key Words

**Conversion graph:** A graph which converts between two variables.
**Intercept:** Where two graphs cross.
**y-intercept:** Where a graph crosses the y-axis.
**Gradient:** The rate of change of one variable with respect to another. This can be seen by the steepness.
**Simultaneous:** At the same time.

## Tip

The solution to two linear equations with two unknowns is the coordinates of the intercept (where they cross).

## Examples
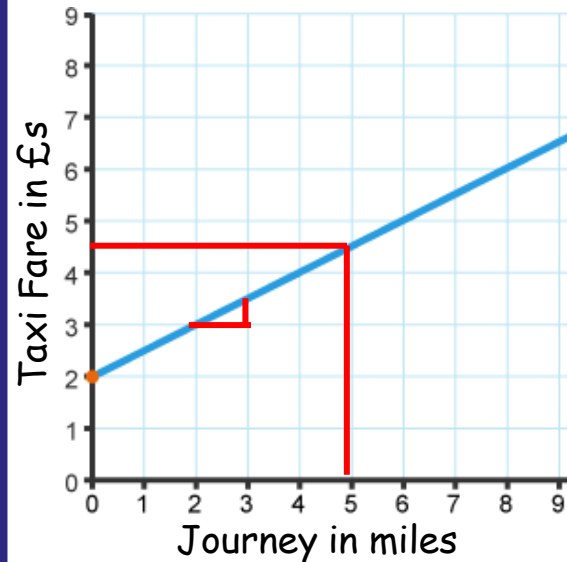


What is the minimum taxi fair?
**£2,** this is the y-intercept.

What is the charge per mile?
**50p,** every extra mile adds on 50p.

How much would a journey of 5 miles cost?
**£4.50,** See line drawn up from 5 miles to the graph, then drawn across to find the cost.

## Questions

1) For the graph above    a) A journey is 8 miles, what is its cost?
b)  A journey cost just £3, how far was the journey?
2) Draw a graph to show the exchange rate   £1 = $1.4.

ANSWERS: 1) a) £6      b) 2 miles

# Half term 3

# FRACTIONS, DECIMALS AND PERCENTAGES

## Key Concepts

A **fraction** is a numerical quantity that is not a whole number.

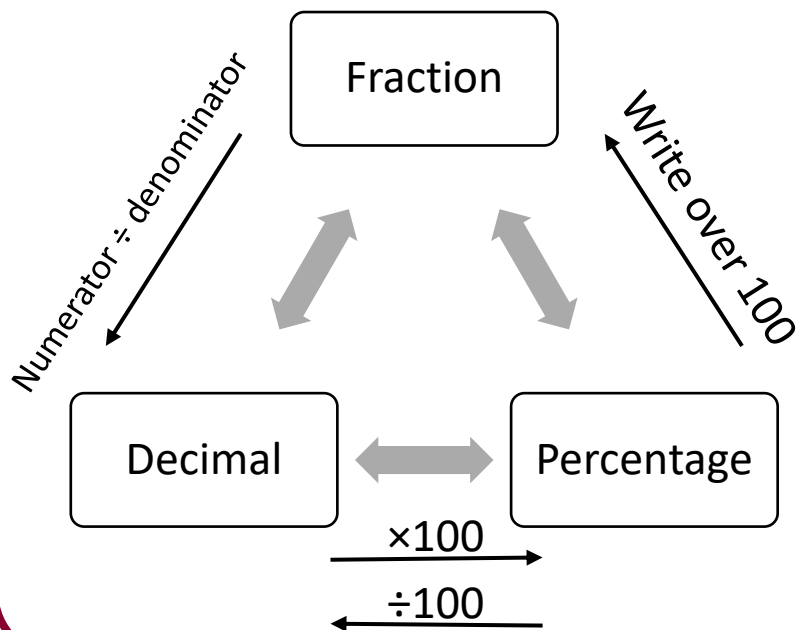A **decimal** is a number written using a system of counting based on the number 10.

| Thousands | Hundreds | Tens | Ones | . | Tenths | Hundredths | Thousandths |
|---|---|---|---|---|---|---|---|
| 8 | 7 | 6 | 5 | . | 4 | 3 | 2 |

A **percentage** is an amount out of 100.

## Key Words
Fraction
Decimal
Percentage
Division
Multiply

## Examples

Fraction

Numerator ÷ denominator

Write over 100

Decimal    ×100    Percentage
           ÷100

Order the following in ascending order:

$\frac{3}{5}$    62%    0.67    $\frac{7}{10}$    0.665

×20    →    ×100    ×10    ×100

$\frac{60}{100}$        $\frac{70}{100}$

60%    62%    67%    70%    66.5%

$\frac{3}{5}$    62%    0.665    0.67    $\frac{7}{10}$

1) Convert the following into percentages:
   a) 0.4   b) 0.08   c) $\frac{6}{20}$   d) $\frac{3}{25}$

2) Compare and order the following in ascending order:

   $\frac{3}{4}$    76%    0.72    $\frac{4}{5}$    0.706

# FRACTIONS

## Key Concepts

$$\frac{x}{y} \longrightarrow \frac{\text{Numerator}}{\text{Denominator}}$$

**Equivalent fractions** have the same value as one another.

Eg. $\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$

## Examples

Calculate $\frac{4}{5}$ of 65:

$65 \div 5 = 13$    Divide by the denominator

$13 \times 4 = 52$    Multiply this by the numerator

$\frac{4}{5}$ of a number is 52, what is the original number?

$52 \div 4 = 13$    Divide by the numerator

$13 \times 5 = 65$    Multiply this by the denominator

Order these fractions in ascending order:

$$\frac{2}{5} \qquad \frac{1}{2} \qquad \frac{5}{6} \qquad \frac{7}{15}$$

$\downarrow \times 6 \quad \downarrow \times 15 \quad \downarrow \times 5 \quad \downarrow \times 2$

$$\frac{12}{30} \qquad \frac{15}{30} \qquad \frac{25}{30} \qquad \frac{14}{30}$$

①      ③      ④      ②

To be able to compare fractions we must have a **common denominator**

**hegartymaths**

**61, 63-70**

## Key Words
**Fraction**
Equivalent
Reciprocal
Numerator
Denominator

1) Calculate $\frac{2}{7}$ of 56.

2) $\frac{3}{8}$ of a number is 36, what is the original number?

3) Order the following in ascending order:    $\frac{2}{3}$    $\frac{5}{6}$    $\frac{3}{8}$    $\frac{7}{12}$

ANSWERS  A 1) 16  2) 96  3) $\frac{3}{8}$  $\frac{7}{12}$  $\frac{2}{3}$  $\frac{5}{6}$

# CONVERSION OF METRIC UNITS

## Key Concept

Metric units of **length**:
*mm, cm, m, km*

Metric units of **weight**:
*g, kg*

Metric units of **capacity**:
*ml, l*

All of these units are **metric** units. They will always use conversions of multiples of 10, eg.10, 100, 1000 etc.
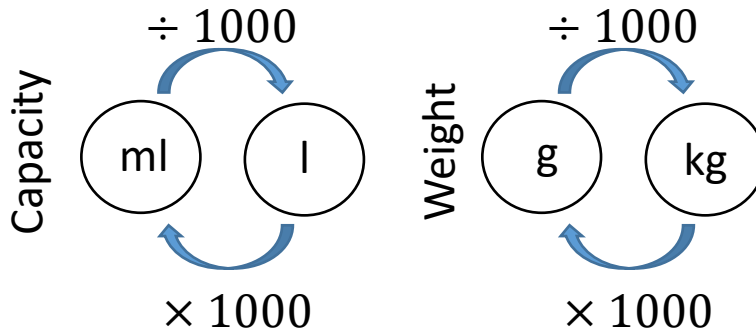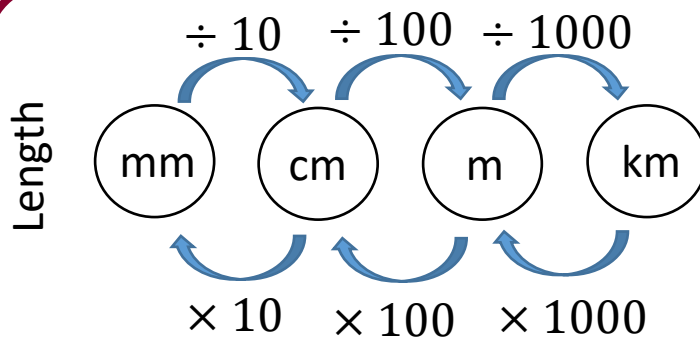
### Length

$\div 10$   $\div 100$   $\div 1000$

mm   cm   m   km

$\times 10$   $\times 100$   $\times 1000$

### Capacity

$\div 1000$

ml   l

$\times 1000$

### Weight

$\div 1000$

g   kg

$\times 1000$

### Converting areas

$\div 100^2$

1m
1m
Area = 1m$^2$

100cm
100cm
Area = 10000cm$^2$

$\times 100^2$

### Converting volumes

$\div 100^3$

1m
1m
1m
Volume = 1m$^3$

100cm
100cm
100cm
Volume = 1000000cm$^3$

$\times 100^3$
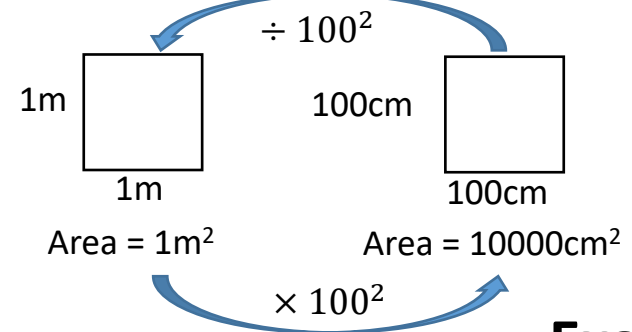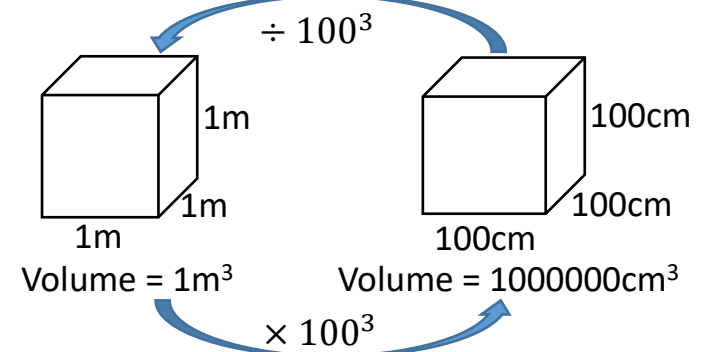
**Examples**

## Key Words
Length
Weight
Capacity
Metric

Convert each of the following:
a) 12cm into mm
b) 1783g into kg
c) 2.5 litres into ml
d) 6.8m into mm
e) 5000000cm$^3$ into m$^3$
f) 2m$^2$ into cm$^2$

ANSWERS: a) 120mm b) 1.783kg c) 2500ml d) 6800mm e) 5m$^3$ f) 20000cm$^2$

# REVIEW OF RATIO

## Key Concept

2 parts → **2:6** ← 6 parts

$=$

**1:3**

$=$

$= \dfrac{1}{4}$

**Clip Numbers**

**328 – 335**
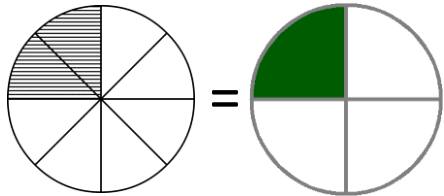
## Key Words

**Ratio:** Relationship between two numbers.
**Part:** This is the numeric value '1' of, would be equivalent to.
**Simplify:** Divide both parts of a ratio by the same number.
**Equivalent:** Equal in value.
**Convert:** Change from one form to another.

## Tip

Its often useful to write the letters above the ratio. This helps you keep the order the correct way round.

## Examples

Simplify   60 : 40 : 100

This could have been done in one step by dividing by 20.

$\div 10$

6 : 4 : 10

$\div 2$

3 : 2 : 5

Write 2 : 5 in the form 1 : n

2 : 5

$\div 2$          $\div 2$

1 : 2.5

Share £45 in the ratio 2 : 7

$45 \div 9 = 5$

£10 : £35

2 : 7

| 5 | 5 |
| 5 | 5 |
| =10 | 5 |
| | 5 |
| | 5 |
| | 5 |
| | 5 |

=35

Joy and Martin share money in the ratio 2 : 5. Martin gets £18 more than Joy. How much do they each get?

$18 \div 3 = 6$

£12 : £30

2 : 5

| 6 | 6 |
| 6 | 6 |
| | 6 |
| | 6 |
| | 6 |

=12  =30

## Questions

1) Simplify   a) 45 : 63   b) 66 : 44   c) 320 : 440
2) Write in the form 1 : n   a) 5 : 10   b) 4 : 6   c) $x : x^2 + x$
3) Share 64 in the ratio 3 : 5   4) Write the ratio 1 : 4 as a fraction.
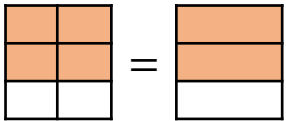
# PROPORTION

## Key Words

**Ratio:** Relationship between two numbers.

**Scale:** The ratio of the length in a drawing to the length of the real thing.

**Proportion:** A name we give to a statement that two ratios are equal.

**Exchange rate:** The value of one currency for the purpose of conversion to another.

## Examples

Write 2: 5 in the form 1 : n

2 : 5

÷ 2            ÷ 2

1 : 2.5

---
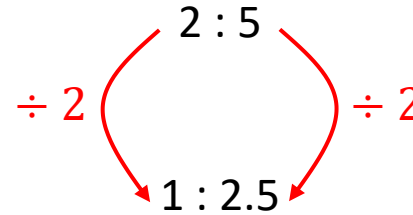
a:b = 4:5 and b:c = 6:7

Find a:b:c.

The LCM of 5 and 6 is 30

a  :  b  :  c

4  :  5

×6            6  :  7        × 5

**24 : 30 : 35**

Cake recipe for 6 people.

3 eggs
300g flour
150g sugar

What would you need for 8 people?

÷ 3        ×4

|        | **6** | **2** | **8** |
|--------|-------|-------|-------|
| eggs   | 3     | 1     | 4     |
| flour  | 300g  | 100g  | 400g  |
| sugar  | 150g  | 50g   | 200g  |

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**331-340, 707-708, 839-842, 864-871**

## Tip

Working with ratio or proportion requires multiplying or dividing the numbers. Do not add or subtract.

## Questions

1) Write in the form 1 : n    a) 4 : 8     b) 3 : 12     c) 4 : 6
2) a : b = 3 : 10 and b : c = 4 : 12.  Find a:b:c.
3) Pancakes for 4 people need 2 eggs, 120g flour and 60ml milk. How much for 6 people?

ANSWERS: 1) a) 1:2 b) 1:4 c) 1:1.5 2) 12:40:120 3) 3 eggs, 180g flour, 90 ml milk.

# Half term 4

# AREA AND PERIMETER

## Key Concepts
### Area

$A = l \times w$
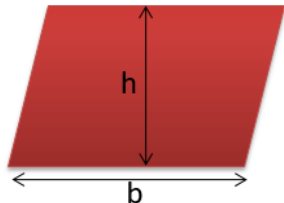
$A = b \times h$
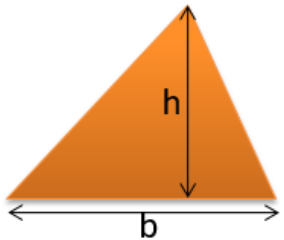
$A = \frac{1}{2}(b \times h)$
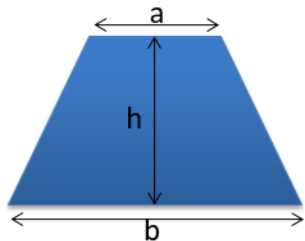
$A = \frac{1}{2}(a+b)h$

## Examples

4 m
2 m

$Area = 2 \times 4 = 8m^2$

$Area = \frac{1}{2}(10 \times 12)$

12 cm

10 cm

$= 60cm^2$

5 mm
14 mm

$Area = 5 \times 14 = 70mm^2$

8cm
2cm
5cm
5 cm
3cm

$Area = (5 \times 3) + (2 \times 5) = 25cm^2$

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## Clip Numbers
### 554 – 559
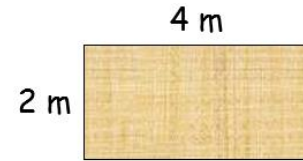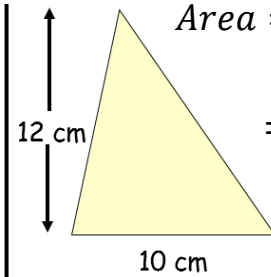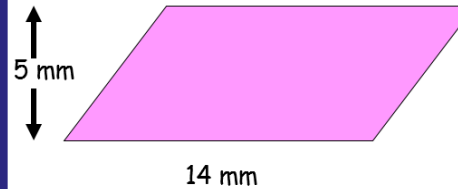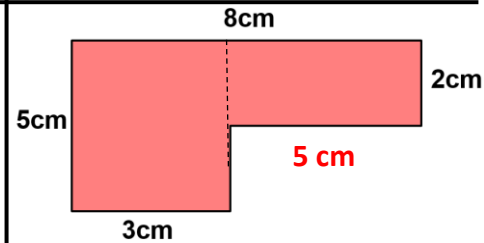
### Tip
Always remember units. These units are squared for area.
$mm^2$, $cm^2$, $m^2$, etc

### Questions – Find the area.

1)
12 cm
8 cm

2)
16 cm
14 cm

3)
7cm
6cm
9cm
15cm

ANSWERS: 1) 96 $cm^2$    2) 112 $cm^2$    2) 87 $cm^2$
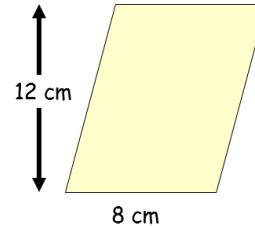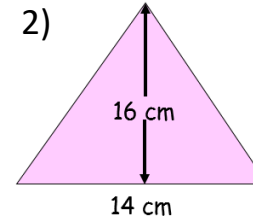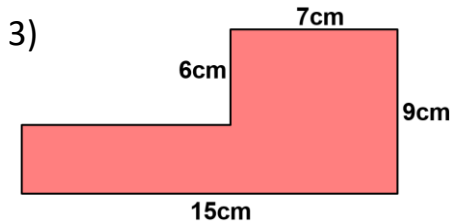
# CIRCLES AND COMPOUND AREA

## Key Concepts



Circumference, radius, diameter, Sector, Tangent

chord, Segment, Arc

## Clip Numbers
### 534-547, 556, 9
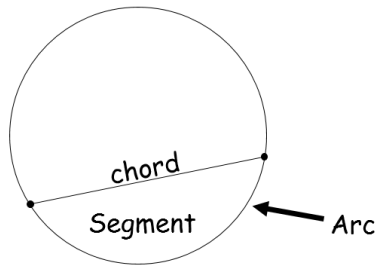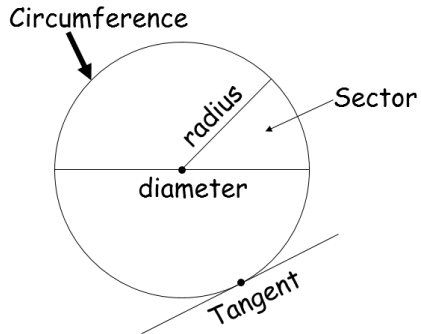
## Key Words

**Diameter:** Distance from one side of the circle to the other, going through the centre.

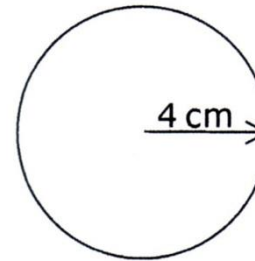**Radius:** Distance from the centre of a circle to the circumference.

**Chord:** A line that intersects the circle at two points.

**Tangent:** A line that touches the circle at only one point.

**Compound (shape):** More than one shape joined to make a different shape.

## Examples

Find the area and circumference to 2dp.



4 cm

$$Circumference = \pi \times d$$
$$= \pi \times 8 = 25.13cm$$
$$Area = \pi \times r^2$$
$$= \pi \times 4^2 = 50.27cm^2$$

Find shaded area to 2dp.



10m
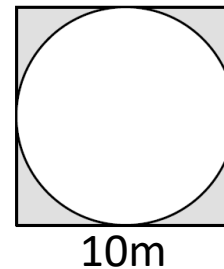
$$Square\ area = 10 \times 10$$
$$= 100m^2$$
$$Circle\ area = \pi \times r^2$$
$$= \pi \times 5^2$$
$$= 78.54\ m^2$$
$$Shaded\ area = 100 - 78.54 = 21.46m^2$$

## Tip
If you don't have a calculator you can leave your answer in terms of $\pi$.

## Questions
1) Find to 1dp the area and circumference of a circle with:

a) Radius = 5cm    b) Diameter = 12mm    c) Radius = 9m

2) Find the area & perimeter of a semi-circle with diameter of 15cm.

## Formula
$$Circle\ Area = \pi \times r^2$$
$$Circumference = \pi \times d$$

ANSWERS: 1) a) A = 78.5cm², C = 31.4cm  b) A = 113.1mm², C = 37.7mm  c) A = 254.5m², C = 56.5m    2) A = 88.4cm²,  P = 38.6cm
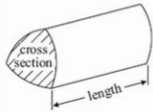
# VOLUME AND SURFACE AREA OF PRISMS

## Key Concept

The **volume** of an object is the amount of space that it occupies. It is measured in units cubed e.g. $cm^3$.

To calculate the volume of any prism we use:
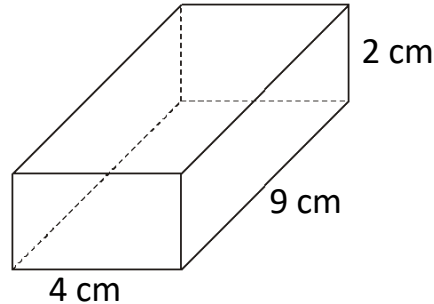
$$\frac{area\ of}{cross\ section} \times length$$

A **prism** is a 3D shape which has a continuous cross-section.

The **surface area** of an object is the sum of the area of all of its faces. It is measured in units squared e.g. $cm^2$.

## Examples

$$Volume = 4 \times 9 \times 2$$
$$= 72cm^3$$

**Surface area**:
$$Front = 4 \times 2 = 8$$
$$Back = 4 \times 2 = 8$$
$$Side\ 1 = 9 \times 2 = 18$$
$$Side\ 2 = 9 \times 2 = 18$$
$$Bottom = 4 \times 9 = 36$$
$$Top = 4 \times 9 = 36$$
$$Total = 124cm^2$$

$$Area\ of\ triangle = \frac{5 \times 7}{2}$$
$$= 17.5mm^2$$

$$Volume = 17.5 \times 11$$
$$= 192.5mm^3$$
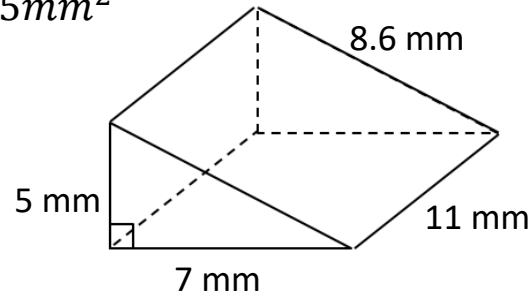
**Surface area**:
$$Front = \frac{7 \times 5}{2} = 17.5$$
$$Back = \frac{7 \times 5}{2} = 17.5$$
$$Side = 5 \times 11 = 55$$
$$Bottom = 7 \times 11 = 77$$
$$Top = 11 \times 8.6 = 94.6$$
$$Total = 261.6cm^2$$

2 cm, 9 cm, 4 cm

8.6 mm, 5 mm, 7 mm, 11 mm
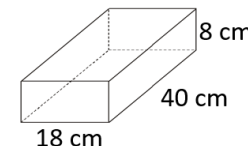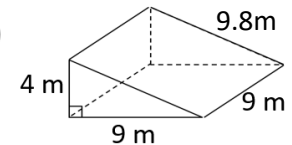
## Key Words
Volume
Capacity
Prism
Surface area
Face

Find the volume and surface area of each of these prisms:

1) 8 cm, 40 cm, 18 cm

2) 9.8m, 4 m, 9 m, 9 m

ANSWERS: 1)Volume = 5760 cm³ Surface area = 2368 cm² 2) Volume = 162 m³ Surface area = 241.2m²
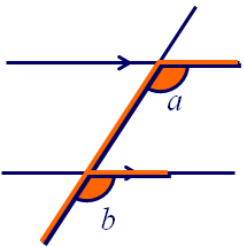
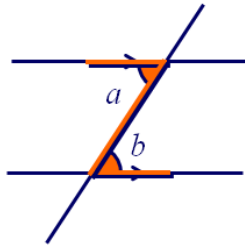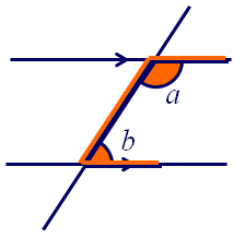# PARALLEL LINES AND ANGLES

## Key Concepts



**Corresponding** angles are equal.    **Alternate** angles are equal.



**Co-interior** angles add to 180°.
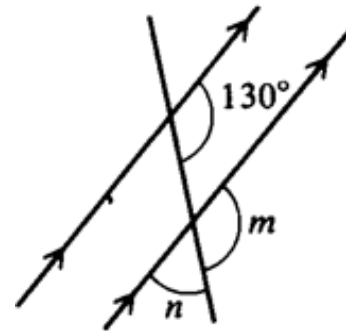
## Key Words

**Intersect:** Two lines which cross.

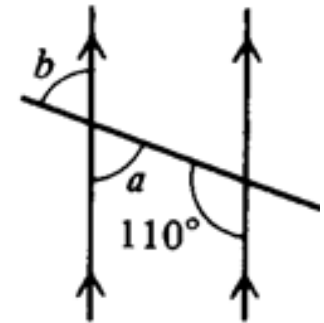**Parallel:** Two lines which never intersect. Marked by an arrow on each line.

**Transversal:** A line which intersects two parallel lines.

## Examples



m = 130° as corresponding angles are equal.
n = 50° as angles on a line add to 180°



a = 70° as co-interior angles add to 180°
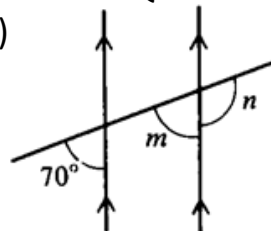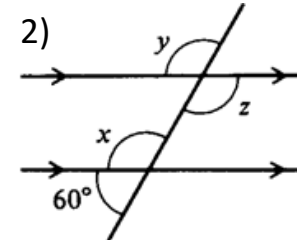b = 70° as vertically opposite angles are equal

## Tip
These angle properties can be used alongside all the other angle properties that you have learnt.

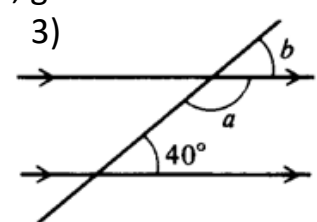**Questions –** Find the labelled angles, give reasons.

1)



2)



3)



ANSWERS: 1) m = 70°, n = 110° 2) x = 120°, y = 120°, z = 120° 3) a = 140°, b = 40°

# Half term 5

# TYPES OF ANGLE AND ANGLES IN POLYGONS

## Key Concepts

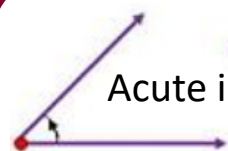**Regular polygons** have equal lengths of sides and equal angles.

**Angles in polygons**

Sum of interior angles
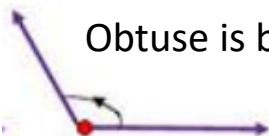$$= (number\ of\ sides - 2) \times 180$$

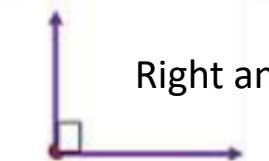Exterior angles of **regular** polygons = $\dfrac{360}{number\ of\ sides}$

**Types of angle**

There are four types which need to be identified – acute, obtuse, reflex and right angled.

## Examples

Acute is less than 90º

Obtuse is between 90º and 180º

Right angled is 90º
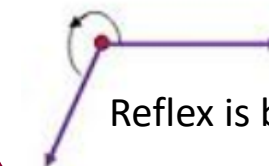
Reflex is between 180º and 360º

### Regular Pentagon
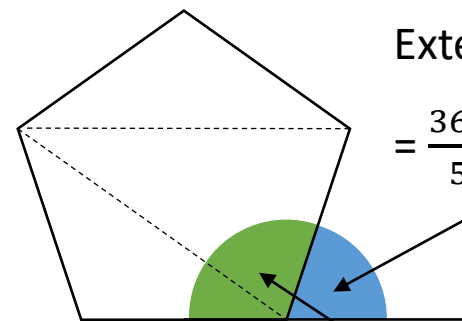
Exterior angles

$$= \frac{360}{5} = 72^o$$

Sum of interior angles
$$= (5 - 2) \times 180$$
$$= 540^o$$

Interior angle $= \dfrac{540}{5} = 108^o$

455, 456,
560-564

**Key Words**
Polygon
Interior angle
Exterior angle
Acute
Obtuse
Right angle
Reflex

## Questions
1) Calculate the sum of the interior angles for this regular shape.
2) Calculate the exterior angle for this regular shape.
3) Calculate the size of one interior angle in this regular shape.

ANSWERS: 1) 720º   2) 60º   3) 120º

# CO-ORDINATES AND TRANSFORMATIONS

## Key Concept

### Reflection



### Rotation



### Translation



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## Clip Numbers
### 199, 205, 637-657
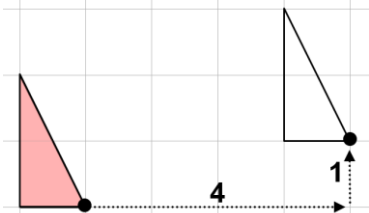
## Key Words

**Co-ordinate:** A pair of numbers which describe the position on a grid.
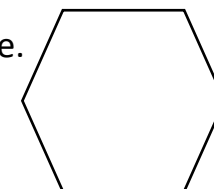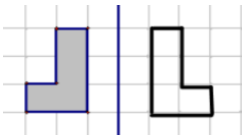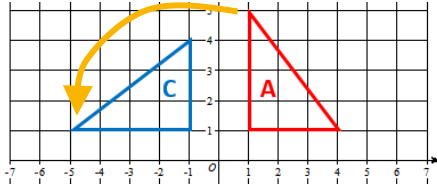**Transformation:** This means the shape has 'changed'.
**Reflection:** This means a shape has been flipped.
**Rotation:** This means a shape has been turned.
**Translation:** This means a *movement* of the shape.

## Tip
- Use **tracing paper** to avoid mistakes.
- When describing transformations, look at how many marks are available and see if you have put enough to get the marks.

## Examples

a) Reflect A in the x-axis, label it B.

b) Rotate A 90°, anti-clockwise about (0,0), label it C.

c) Translate A in the vector $\begin{pmatrix} -7 \\ -6 \end{pmatrix}$, label it D.



## Questions

Draw a grid like the one above.
Plot a triangle with vertices (6,2), (3, 2) and (4, 5).

a) Reflect the triangle in the y-axis.   b) Translate the triangle $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

ANSWERS:   a) (-6,2), (-3,2) and (-4,5)       b) (1,1), (0,-2) and (3,-2)

# ENLARGEMENT

## Key Concepts

An **enlargement** changes the size of an image using a scale factor from a given point.

A **positive scale factor** will increase the size of an image.

A **fractional scale factor** will reduce the size of an image.

## Examples

Enlarge shape A by scale factor 2 from point P.
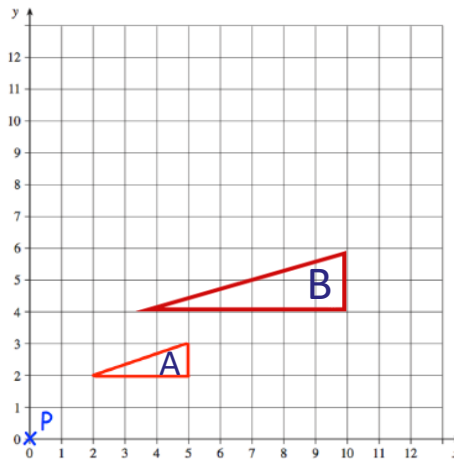


Enlarge by scale factor $\frac{1}{2}$ from point P.
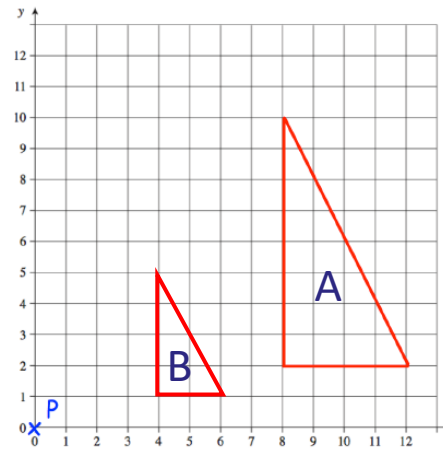
**637,638,650, 642-645, 651**

**Key Words**
**Enlargement**
**Scale factor**
**Centre**
**Positive**

Describe the **single** transformation you see on each coordinate grid from A to B:



ANSWERS: a) enlarge, centre (-4,2) scale factor 2   b) enlarge, centre (1,-2) scale factor $\frac{1}{2}$

# REVIEW OF PROBABILITY

## Key Concept

### Chance

Impossible — Unlikely — Even Chance — Likely — Certain

### Probability

| 0 | 0.25 | 0.5 | 0.75 | 1 |
|---|------|-----|------|---|
| 0% | 25% | 50% | 75% | 100% |
| 0 | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | 1 |

Probabilities can be written as:
- Fractions
- Decimals
- Percentages



## hegartymaths

### Clip Numbers

### 349 - 359

## Key Words

**Probability:** The chance of something happening as a numerical value.

**Impossible:** The outcome cannot happen.

**Certain:** The outcome will definitely happen.

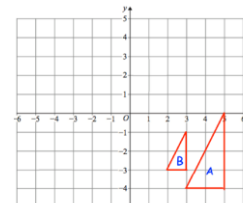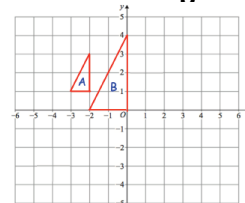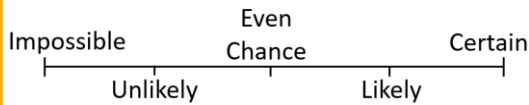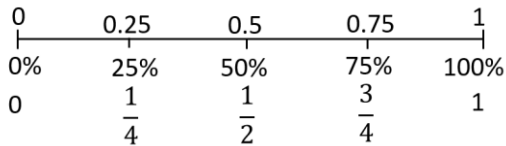**Even chance:** The are two different outcomes each with the same chance of happening.

**Expectation:** The amount of times you expect an outcome to happen based on probability.

## Tip
Probabilities always add up to 1.

## Formula
*Expectation*
$= Probability \times no. of \ trials$

## Examples
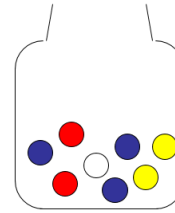
1) What is the probability that a bead chosen will be **yellow**.
Show the answer on a number line.

$$Probability = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$$

$$P(Yellow) = \frac{2}{8} = \frac{1}{4}$$

0 — $\frac{1}{8}$ — $\frac{2}{8}$ — $\frac{3}{8}$ — $\frac{4}{8}$ — $\frac{5}{8}$ — $\frac{6}{8}$ — $\frac{7}{8}$ — 1
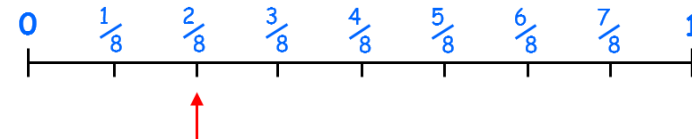
2) How many **yellow** beads would you **expect** if you pulled a bead out and replaced it 40 times?

$$\frac{1}{4} \times 40 = \frac{1}{4} of\ 40 = 10$$

## Questions
In a bag of skittles there are 12 red, 9 yellow, 6 blue and 3 purple left.
Find:  a)  P(Red)  b) P(Yellow)  c) P(Red or purple)    d) P(Green)

ANSWERS: 1) a) $\frac{12}{30} = \frac{2}{5}$  b) $\frac{9}{30} = \frac{3}{10}$  c) $\frac{15}{30} = \frac{1}{2}$  d) 0

# FURTHER PROBABILITY

## Key Words

**Probability:** The chance of something happening as a numerical value.
**Impossible:** The outcome cannot happen.
**Certain:** The outcome will definitely happen.
**Even chance:** The are two different outcomes each with the same chance of happening.
**Mutually Exclusive:** Two events that cannot both occur at the same time.

## Examples

In Hannah's class there are 32 students.
15 of these students are boys.
7 of the boys have a pet.
9 girls do not have a pet.

Have a pet — 7
15
boys
Do not have a pet — 8
32
Have a pet — 8
girls
17
Do not have a pet — 9
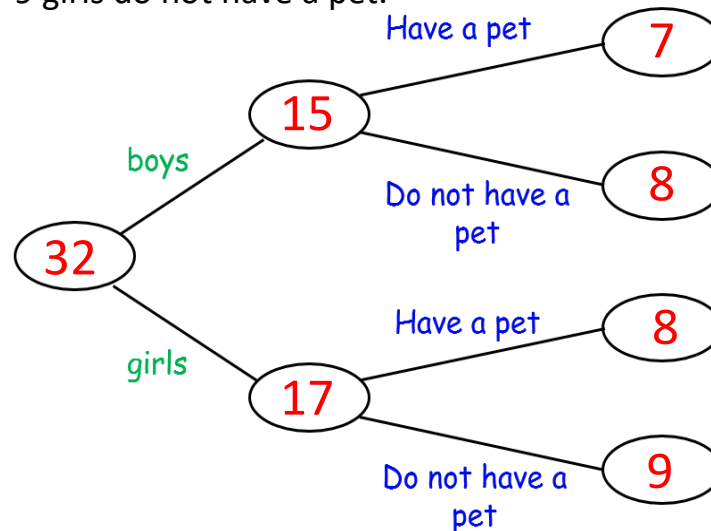
$$P(boy) = \frac{15}{32}$$

$$P(Girl\ with\ pet) = \frac{8}{32}$$

## Questions

1) Draw a two-way table for the question above.
2) Find the probability that a pupil chosen is a boy with no pets.
3) A girl is chosen, what is the probability she has a pet?

ANSWERS:   2) $\frac{8}{32}$   3) $\frac{8}{17}$

# Half term 6

# TWO WAY TABLES

## Key Concept

A **two way table** is used to represent categorised data.

## Examples

This **two way table** gives information on how 100 students travelled to school.

| | Walk | Car | Other | Total |
|---|---|---|---|---|
| **Boy** | 15 | 25 | 14 | 54 |
| **Girl** | 22 | 8 | 16 | 46 |
| **Total** | 37 | 33 | 30 | 100 |

Always double check that your rows and columns add up to the total value.

## Key Words
**Two way table**
**Compare**

Complete a two way table using this information:
Felicity asked 100 students how they came to school one day. Each student walked or came by bicycle or came by car.
49 of the 100 students are girls.
10 of the girls came by car.
16 boys walked.
21 of the 41 students who came by bicycle are boys.
Work out the total number of students who walked to school.

# PIE CHARTS AND SCATTER-GRAPHS

## Key Concepts

**Pie charts** use angles to represent proportionally the quantity of each group involved.

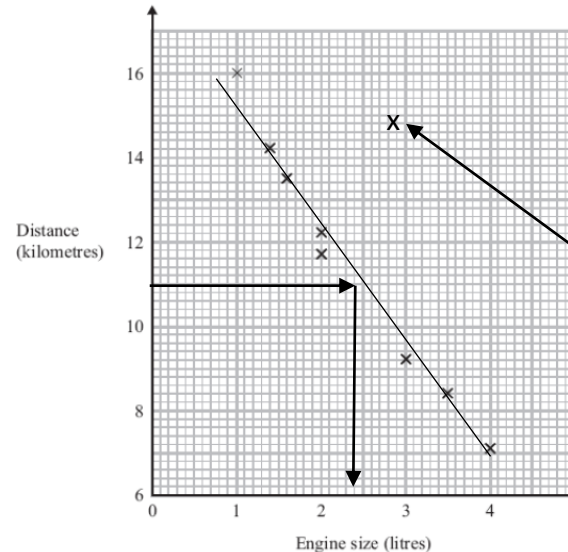Pie charts can only be compared to one another when populations are given.

**Scatter-graphs** show the relationship between two variables. This relationship is called the **correlation**.



Positive | Negative | No Correlation
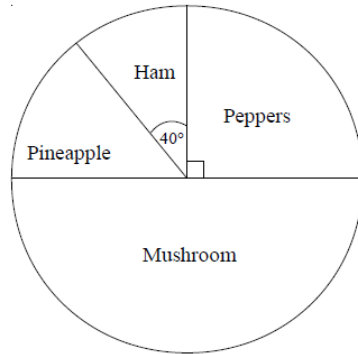
## Examples



| Topping | Frequency | Angle of Sector |
|---|---|---|
| Peppers | 18 | 90° |
| Mushroom | 36 | 180° |
| Pineapple | 10 | 50° |
| Ham | 8 | 40° |
| | 72 | 360° |

$$\frac{360}{72} = 5$$

$\times 5$



A scatter-graph is drawn to show the relationship between the engine size of a car and how far it can travel.

This graph shows negative correlation.

This is an outlier.

We draw a line of best fit through the middle of the data points to read from to estimate readings. For example, estimating the engine size of a car that can travel 11km would be 2.5 litres.

**Key Words**
**Pie chart**
**Scatter-graph**
**Correlation**
**Outlier**
**Variable**

1) Calculate the angle for each category:

| Region | Frequency |
|---|---|
| Southern England | 9 |
| London | 23 |
| Midlands | 16 |
| Northern England | 12 |
| Total | 60 |
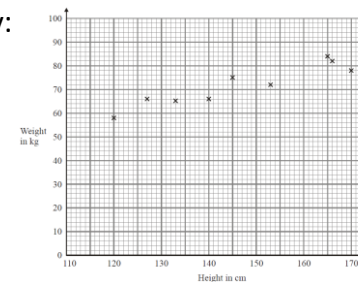


2a) What type of correlation is shown?
b) Using a line of best fit estimate the weight when the height is 135cm.

ANSWERS: 1) 54, 138, 96, 72  2) a) positive b) 64kg-66kg

# PRESENTING AND INTERPRETTING DATA

## Key Concept
### Pie Charts

There are 360 degrees in a pie chart. So you need angles that add to 360°.

| Eye colour | F |
|---|---|
| Blue | 15 | × 4 = 60 |
| Brown | 43 | × 4 = 172 |
| Other | 32 | × 4 = 128 |

$\frac{360}{90} = 4$        = 90        = 360

## Key Words

**Frequency:** Total.
**Mean:** Total of data divided by the number of pieces of data.
**Mode:** The value that occurs most frequently.
**Median:** Middle number when they are in order.
**Range:** Difference between the largest and smallest values.

## Tips
- There can be more than one mode.
- Range is a measure of spread, not an average.
- Bar charts have gaps between the bars.

## Examples

5, 9, 9, 9, (11) 12, 13, 15, 16

**Averages**

$$Mean = \frac{5 + 9 + 9 + 9 + 11 + 12 + 13 + 15 + 16}{9} = \frac{99}{9} = 11$$

Median = 11    (The middle number shown above)

Mode = 9    (This number occurs most often)

### Measure of Spread

Range = 16 − 5 = 11
    (A bigger range means the data is more spread out)

## Questions
1) Find the mean, mode, median and range of:
        a)    3, 12, 4, 6, 8, 5, 4      b) 12, 1, 10, 1, 9, 3, 4, 9, 7, 9
2) For the table:
a)    Draw a pie chart to show the data.
b)    Draw a bar chart to show the data.
c)    Work out the mean of the data.

| Age | Frequency |
|---|---|
| 11 | 17 |
| 12 | 11 |
| 13 | 8 |

ANSWERS: 1) a) Mean = 6, Mode = 4, Median = 5, Range = 9        b) Mean = 6.5, Mode = 9, Median = 8, Range = 11        2) a) Angles 170°, 110°, 80°    c) 11.75