

Year 7 Maths Knowledge Organiser (H)

Half Term 1

PLACE VALUE, DECIMALS & USING SCALES

Key Concept

Multiply/Divide by powers of 10

10 000	1000	100	10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
					•			

Multiplying

X 10
X 100
X 1000

digits move LEFT 1 space
digits move LEFT 2 spaces
digits move LEFT 3 spaces



Dividing

÷ 10
÷ 100
÷ 1000

digits move RIGHT 1 space
digits move RIGHT 2 spaces
digits move RIGHT 3 spaces



Key Words

Decimal: A number that contains a point.

Metric measure: The unit used to measure length, mass etc.

Scale: The conversion to convert between drawings and real life sizes.

Examples

Ordering Decimals

0.3, 0.21, 0.305, 0.38, 0.209

Add zero's so that they all have the same number of decimal places.

0.300, 0.210, 0.305, 0.380, 0.209

Then they can be placed in order:

0.209, 0.21, 0.3, 0.305, 0.38

Multiplying/Dividing by powers of 10

3.4×100

100	10	1	•	$\frac{1}{10}$
		3	•	4
3	4	0	•	

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13-16, 46, 691,
864

Tip

- Add digits when ordering decimals.
- The number of zero's tells you the number of places to move the digits.

Questions

- Order 1.52, 1.508, 1.5, 1.05, 1.51
- Work out a) 1.35×10 b) 0.6×100 c) $4.5 \div 100$
- Convert a) 36 mm to cm b) 7 cm to mm c) 450 cm to m
d) 620 g to kg e) 4.2 kg to g f) 0.7 kg to g

ANSWERS: 1) 1.05, 1.5, 1.508, 1.51, 1.52 2) a) 13.5 b) 60 c) 0.045
3) a) 3.6cm b) 70mm c) 4.5m d) 0.62kg e) 4200 f) 700g

INTEGERS, ROUNDING AND PLACE VALUE

Key Concepts

Digits are the individual components of a number.

Integers are whole numbers.

Rounding rules:

A value of 5 to 9 rounds the number up.

A value of 0 to 4 keeps the number the same.

Examples

Order the following numbers starting with the smallest:

1) 5, -3, 4, 7, -2
-3, -2, 4, 5, 7

2) 0.067 0.6 0.56 0.65 0.605
Rewrite 0.067, 0.600, 0.560, 0.650, 0.605
0.067 0.56 0.6 0.605 0.65

Round 3.527 to:

a) 1 decimal place

3.5 **2** 7 → 3.5

b) 2 decimal places

3.5 **2** **7** → 3.53

c) 1 significant figure

3 **5** 2 7 → 4

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1 – 3, 31 – 32

Key Words

Integer Even
Digit Odd
Decimal place
Significant figures

A) Order the following numbers starting with the smallest:

1) 6, -2, 0, -5, 3 2) 0.72, 0.7, 0.072, 0.07, 0.702

B) Round the following numbers to the given degree of accuracy

1) 14.1732 (1 d.p.) 2) 0.0568 (2 d.p.) 3) 3418 (1 S.F)

ANSWERS: A1) -5, -2, 0, 3, 6 2) 0.07, 0.072, 0.7, 0.702, 0.72 B1) 14.2 2) 0.06 3) 3000

DECIMALS

Key concepts

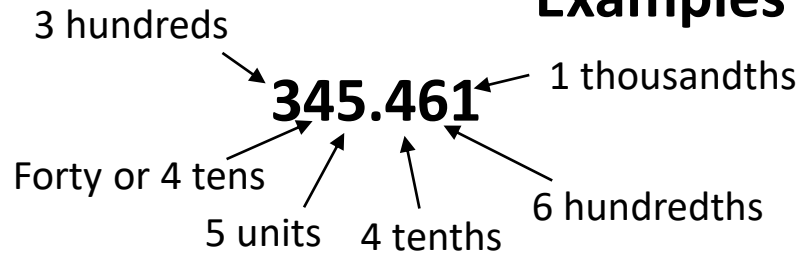
Place value:

Th H T U . t h th

When adding and subtracting decimals we must ensure the decimal places are underneath each other when setting up.

When multiplying decimals, calculate without the decimal point and use estimation to help replace it.

Examples



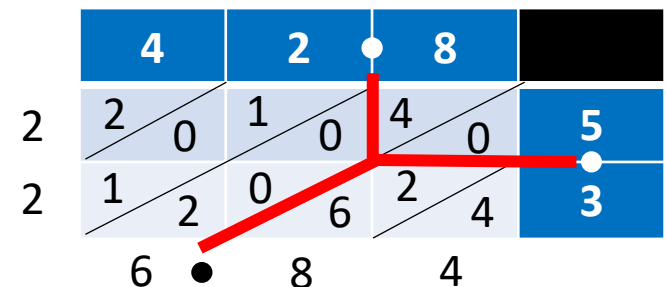
$$42.8 + 5.32$$

$$\begin{array}{r} 42.80 \\ + 5.32 \\ \hline 48.12 \end{array}$$

$$42.8 - 5.32$$

$$\begin{array}{r} 42.80 \\ - 5.32 \\ \hline 37.48 \end{array}$$

$$42.8 \times 5.3$$



$$226.84$$

Estimated answer $40 \times 5 = 200$

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102 - 110

Key Words

Decimal
Tenths
Hundredths
Thousandths

A) What is the value of the 4 in each number?

1) 498 2) 8746 3) 6.243 4) 1.004

B) Work out:

1) $3.1 + 5.27$ 2) $16.4 - 9.18$ 3) 0.03×500 4) 3.4×5.6





5) 4.79×6.8

ANSWERS: A 1) 4 hundred 2) forty 3) 4 hundredths 4) 4 thousandths
B 1) 8.37 2) 7.22 3) 15 4) 19.04 5) 32.572

FOUR OPERATIONS WITH INTEGERS & DECIMALS

Key Words

Place Value: The value a digit takes when placed in a particular position of a number.

 <p>Add Sum Total All together Plus In all</p>	 <p>Multiply Product Times Twice Total Multiplied by</p>
 <p>Subtract Remain Difference Less than Fewer How many more Minus</p>	 <p>Divide Quotient Goes into Split Equally Each</p>

Examples

$$48 + 36 = 84$$



$$74 - 27 = 47 \text{ worked by counting back:}$$



$$\begin{array}{r} 97 \\ 3 \overline{)292} \end{array}$$

$$\begin{array}{r} 258 \\ + 87 \\ \hline 345 \\ 11 \end{array}$$

$$\begin{array}{r} 315 \\ - 28 \\ \hline 287 \end{array}$$

$$\begin{array}{r} 38 \\ \times 7 \\ \hline 56 \\ 210 \\ \hline 266 \end{array}$$

$$56 \times 27$$

\times	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

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Clip Numbers

1-22, 141-146, 47

Tip

Multiplication and addition are associative, so you can work them out in any order.

So 3×4 is the same as 4×3 .

$4 + 3$ is the same as $3 + 4$

Questions

- a) $49 + 37$ b) $125 + 69$ c) $5.6 + 24.8$
- a) $64 - 28$ b) $134 - 57$ c) $16.2 - 9.5$
- a) 7×146 b) 34×67 c) 2.9×7.2 4) a) $294 \div 7$ b) $192 \div 6$

ANSWERS : 1) a) 86 b) 194 c) 30.4
2) a) 36 b) 77 c) 6.7
3) a) 1022 b) 2278 c) 20.88
4) a) 42 b) 32

ORDER OF OPERATIONS

Key Concept

- B** Brackets
- I** Indices
- D** Division
- M** Multiplication
- A** Addition
- S** Subtraction

If a calculation contains the looped calculations work from left to right.

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24, 39-44, 120,
150, 181-189

Key Words

Operation: In maths these are the functions $\times \div + -$.

Commutative: Calculations are commutative if changing the order does not change the result.

Associative: In these calculations you can re-group numbers and you will get the same answer.

Indices: These are the squares, cubes and powers.

Tip

- Put brackets around the calculations which need to be done first.
- Indices also includes roots.

Examples

$$\underbrace{5 \times 4}_{20} - \underbrace{8 \div 2}_4 = 16$$

$$\begin{aligned} & (2^2 + 6)^2 \times 4 - 8 \\ & \downarrow \\ & (4 + 6)^2 \times 4 - 8 \\ & \downarrow \\ & (10)^2 \times 4 - 8 \\ & \downarrow \\ & 100 \times 4 - 8 \\ & \downarrow \\ & 400 - 8 = 392 \end{aligned}$$

Questions

- 1) $7 - 10 \div 2$
- 2) $4^3 - 13 \times 4$
- 3) $21 \div 7 - 2$
- 4) $12 \div (7 - 3)$
- 5) $20 \div 2^2$
- 6) $(16 - 13) \div 3$
- 7) Place brackets to make the calculation work $20 \div 5 - 3 = 10$

FACTORS, MULTIPLES AND PRIMES

Key Concept

Factors:

Find these in pairs

12

1, 12

2, 6

3, 4

Multiples:

Start with the number itself

7 – 7, 14, 21, 28, ...

Key Words

Factor: The numbers which fit into a number exactly.

Multiple: The numbers in the times table.

Prime: Numbers which have only two factors which are 1 and itself.

Highest Common Factor: The highest factor which is common for both numbers.

Lowest Common Multiple: The smallest multiple which is common to both numbers.

Examples

Lowest Common Multiple (LCM)

Q - Find the LCM of 6 and 7:

6 – 6, 12, 18, 24, 30, 36, **42**, 48, 54, 60, ...

7 – 7, 14, 21, 28, 35, **42**, 49, 56, ...

LCM = 42

Highest Common Factor (HCF)

Q – Find the HCF of 18 and 24

18 – 1, 2, 3, **6**, 9, 18

24 – 1, 2, 3, 4, **6**, 8, 12, 24

HCF = 6



Clip Numbers

4, 6, 10, 26 – 34

Tip

There is only one even prime number which is the number 2. This can be used to help solve lots of problems.

Questions

- 1) List the first 5 multiples of: a) 7 b) 12 c) 50
- 2) List the factors of: a) 12 b) 15 c) 16
- 3) a) Find the LCM of 5 and 7 b) Find the HCF of 20 and 16

ANSWERS: 1) a) 7, 14, 21, 28, 35 b) 12, 24, 36, 48, 60 c) 50, 100, 150, 200, 250
2) a) 1, 2, 3, 4, 6, 12 b) 1, 3, 5, 15 c) 1, 2, 4, 8, 16
3) a) 35 b) 4

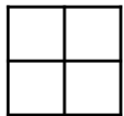
Types of Numbers & Prime Factorisation

Key Concept

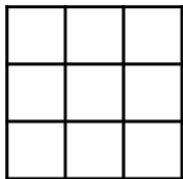
Square numbers



$$1^2 \\ 1 \times 1 = 1$$



$$2^2 \\ 2 \times 2 = 4$$



$$3^2 \\ 3 \times 3 = 9$$

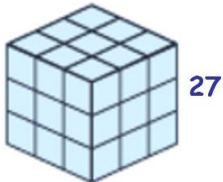
Cube numbers



$$1^3 \\ 1 \times 1 \times 1$$



$$2^3 \\ 2 \times 2 \times 2$$



$$3^3 \\ 3 \times 3 \times 3$$

Key Words

Square: A square number is the result of multiplying a number by itself.

Cube: A cube number is the result of multiplying a number by itself twice.

Root: A root is the reverse of a power.

Prime number: A prime is a number that has only two factors which are 1 and itself.

Factor: A number that fits into another number exactly.

Tip

A number with an odd amount of factors must be a square number.

Examples

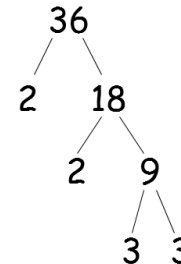
What is 2^4 ?

$$2 \times 2 \times 2 \times 2 = 16$$

What is $\sqrt{64}$?

$$8^2 = 64, \text{ so } \sqrt{64} = \pm 8$$

Write 36 as a product of prime factors



$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

Product means 'multiply'



Clip Numbers
27-30, 99-101

Questions

- 1) a) 2^5 b) 3^3 c) 1^{17} d) $\sqrt{81}$ e) $\sqrt{16}$ f) $\sqrt[3]{64}$
- 2) Write 72 as a product of primes.

ANSWERS: 1) a) 32 b) 27 c) 1 d) ± 9 e) ± 4 f) 4
2) $2^3 \times 3^2$

HCF & LCM USING FACTOR DECOMPOSITION

Key Concepts

Prime factor decomposition

Breaking down a number into its prime factors

Highest common factor

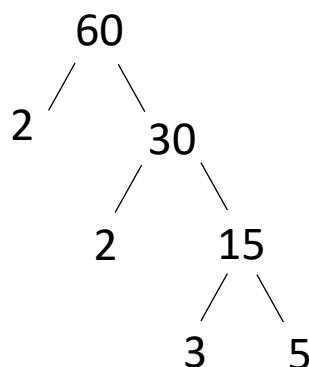
Finding the largest number which divides into all numbers given

Lowest common multiple

Finding the smallest number which both numbers divide into

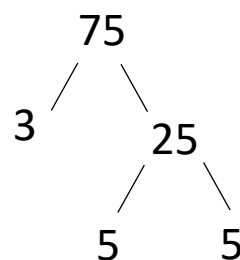
Examples

Find the **highest common factor** and **lowest common multiple** of 60 and 75:



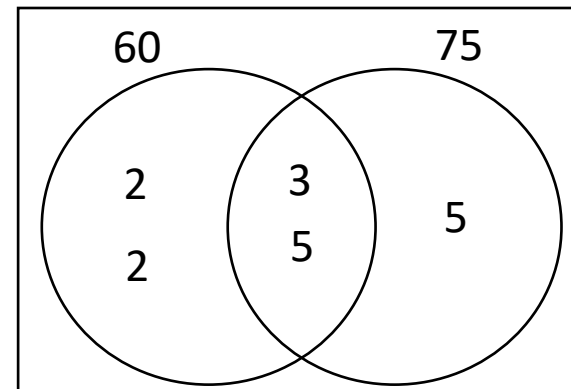
$$2 \times 2 \times 3 \times 5$$

$$2^2 \times 3 \times 5$$



$$3 \times 5 \times 5$$

$$3 \times 5^2$$



HCF – Multiply all numbers in the intersection
 $= 3 \times 5 = 15$

LCM – Multiply all numbers in the Venn diagram
 $= 2 \times 2 \times 3 \times 5 \times 5 = 300$

Key Words

Factor

Multiple

Prime

Highest Common Factor

Lowest Common
Multiple

Questions

- 1) Write 80 as a product of its prime factors
- 2) Write 48 as a product of its prime factors
- 3) Find the LCM and HCF of 80 and 48

Half Term 2

EXPRESSIONS/EQUATIONS/IDENTITIES AND SUBSTITUTION

Key Concepts

A **formula** involves two or more letters, where one letter equals an **expression** of other letters.

An **expression** is a sentence in algebra that does NOT have an equals sign.

An **identity** is where one side is the equivalent to the other side.

When **substituting** a number into an expression, replace the letter with the given value.

Examples

- 1) $5(y + 6) \equiv 6y + 30$ is an identity as when the brackets are expanded we get the answer on the right hand side
- 2) $5m - 7$ is an **expression** since there is no equals sign
- 3) $3x - 6 = 12$ is an **equation** as it can be solved to give a solution
- 4) $C = \frac{5(F - 32)}{9}$ is a **formula** (involves more than one letter and includes an equal sign)
- 5) Find the value of $3x + 2$ when $x = 5$
 $(3 \times 5) + 2 = 17$
- 6) Where $A = b^2 + c$, find A when $b = 2$ and $c = 3$
 $A = 2^2 + 3$
 $A = 4 + 3$
 $A = 7$

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153, 189

Key Words

Substitute
Equation
Formula
Identity
Expression

Questions

- 1) Identify the equation, expression, identity, formula from the list
(a) $v = u + at$ (b) $u^2 - 2as$
(c) $4x(x - 2) = x^2 - 8x$ (d) $5b - 2 = 13$
- 2) Find the value of $5x - 7$ when $x = 3$
- 3) Where $A = d^2 + e$, find A when $d = 5$ and $e = 2$

(d) equation

(c) identity

(b) expression

ANSWERS: 1) (a) formula
(b) expression
(c) identity
(d) equation

2) 8

3) A = 27

ALGEBRAIC EXPRESSIONS

Key Concepts

When collecting like terms involving addition or subtraction, add/subtract the numbers in front of the letters.

If the like terms are multiplied, multiply the numbers in front of the letters and put the letters next to each other.

If the like terms are divided, divide the numbers in front of the letters.



151 – 152, 156 – 157

Key Words

Simplify
Term
Collect

Examples

Simplify the following expressions:

$$1) \quad 4p + 6t + p - 2t = 5p + 4t$$

$$2) \quad 3 + 2t + p - t + 2 = 5 + t + p$$

$$3) \quad f + 3g - 4f = 3g - 3f$$

$$4) \quad f^2 + 4f^2 - 2f^2 = 3f^2$$

$$5) \quad 6a \times 3b \times 2c = 36abc$$

$$6) \quad \frac{9b}{3} = 3b$$

Questions

Simplify:

$$1) \quad 7p + 3q + p - 3q$$

$$3) \quad m - 8g - 5m$$

$$5) \quad 2a \times 5b \times 4c$$

$$7) \quad \frac{36p}{12}$$

$$2) \quad 5 + 4t + 3p - 2t + 7$$

$$4) \quad b^2 - 7b^2 + 2b^2$$

$$6) \quad 8m \times 3n \times 2m$$

$$8) \quad \frac{6t}{18}$$

EXPAND AND SIMPLIFY BRACKETS

Key Concepts

Expanding brackets

Multiply the number outside the brackets with EVERY term inside the brackets

Factoring expressions

Take the highest common factor outside the bracket.



160, 161, 168, 189,
105, 106

Key Words

Expand
Factorise
Simplify

Examples

Expand and simplify where appropriate

1) $7(3 + a) = 21 + 7a$

2) $2(5 + a) + 3(2 + a) = 10 + 2a + 6 + 3a$
 $= 5a + 16$

3) Factorise $9x + 18 = 9(x + 2)$

4) Factorise $6e^2 - 3e = 3e(2e - 1)$

Questions

1) Expand and simplify

(a) $3(2 - 7f)$ (b) $5(m - 2) + 6$ (c) $3(4 + t) + 2(5 + t)$

2) Factorise

(a) $6m + 12t$ (b) $9t - 3p$ (c) $4d^2 - 2d$

SOLVING EQUATIONS

Key Concept

Inverse Operations

Operation	Inverse
+	—
—	+
×	÷
÷	×
x^2	\sqrt{x}

Key Words

Unknown: A letter which represents a number we do not know the value of.

Terms: The numbers and letters in the expression or equation.

Inverse: The operation which will do the opposite.

Examples

$x + 9 = 16$ $-9 \quad -9$ $x = 7$	$x - 12 = 20$ $+12 \quad +12$ $x = 32$	$\frac{x}{3} = 5$ $\times 3 \quad \times 3$ $x = 15$	$2x + 5 = 14$ $-5 \quad -5$ $2x = 9$ $\div 2 \quad \div 2$ $x = 4.5$
--	--	--	--

$\frac{x}{4} - 2 = 4$ $+2 \quad +2$ $\frac{x}{4} = 6$ $\times 4 \quad \times 4$ $x = 24$	$2(3x + 5) = -14$ expand $6x + 10 = -14$ $-10 \quad -10$ $6x = -24$ $\div 6 \quad \div 6$ $x = -4$
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Clip Numbers
177 – 184

Tip

Answers can be:

- Integers
- Decimals
- Fractions
- negatives

Questions

- 1) $x + 8 = 19$ 2) $y - 25 = 15$ 3) $2y = 82$ 4) $\frac{t}{4} = 7$
 5) $\frac{p}{2} - 6 = 2$ 6) $3(2x - 3) = 15$

ANSWERS: 1) $x = 11$, 2) $y = 40$, 3) $y = 41$, 4) $t = 28$, 5) $p = 16$, 6) $x = 4$.

SEQUENCES

Key Concepts

Arithmetic or linear sequences

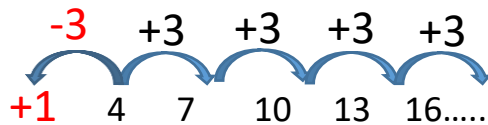
increase or decrease by a common amount each time.

Geometric series has a common multiple between each term.

Quadratic sequences include an n^2 . It has a common second difference.

Fibonacci sequences are where you add the two previous terms to find the next term.

Linear/arithmetic sequence:



a) State the n th term

$3n + 1$
 Difference Go back a term

b) What is the 100th term in the sequence?

$$3n + 1$$

$$3 \times 100 + 1 = 301$$

c) Is 100 in this sequence?

$$3n + 1 = 100$$

$$3n = 99$$

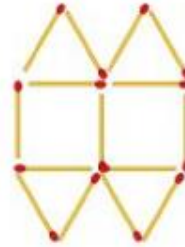
$$n = 33$$

Yes as 33 is an integer.

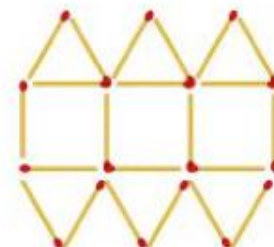
Pattern 1



Pattern 2



Pattern 3



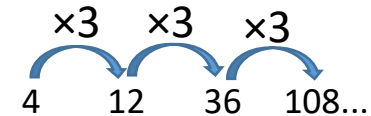
Hint: Firstly write down the number of matchsticks in each image:

$$7n + 1$$

Pattern 1	Pattern 2	Pattern 3
8	15	22

+1
-7 +7 +7

Geometric sequence e.g.



Quadratic sequence e.g.

$n^2 + 4$ Find the first 3 numbers in the sequence

First term: $1^2 + 4 = 5$

Third term: $3^2 + 4 = 13$

Second term: $2^2 + 4 = 8$

Examples

Linear sequences with a picture:

State the n th term.

Key Words

Linear
Arithmetic
Geometric
Sequence
Nth term

1) 1, 8, 15, 22, ...

a) Find the n th term b) Calculate the 50th term c) Is 120 in the sequence?

2) $n^2 - 5$ Find the first 4 terms in this sequence

PLOTTING AND INTERPRETING GRAPHS

Key Concept

Substitution – This is where you replace a number with a letter

If $a = 5$ and $b = 2$

$a + b =$	$5 + 2 = 7$
$a - b =$	$5 - 2 = 3$
$3a =$	$3 \times 5 = 15$
$ab =$	$5 \times 2 = 10$
$a^2 =$	$5^2 = 25$

Key Words

Intercept: Where two graphs cross.

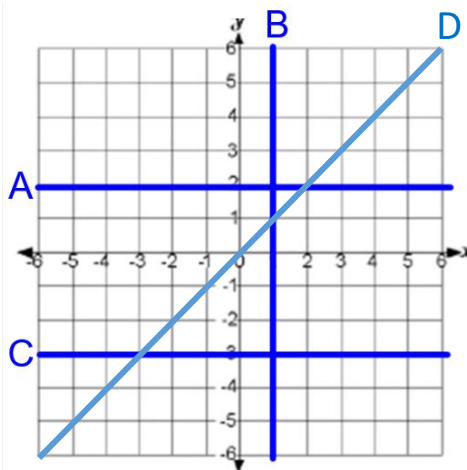
Gradient: This describes the steepness of the line.

y-intercept: Where the graph crosses the y-axis.

Linear: A linear graph is a straight line.

Quadratic: A quadratic graph is curved, u or n shape.

Examples

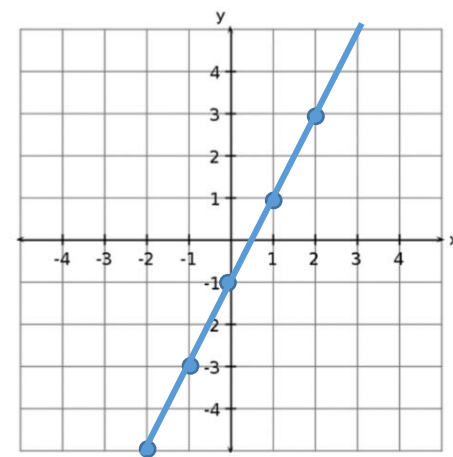


A: $y = 2$ B: $x = 1$

C: $y = -3$ D: $y = x$

Draw the graph of $y = 2x - 1$

X	-2	-1	0	1	2
Y	-5	-3	-1	1	3



Notice this graph has a gradient of 2 and a y-intercept of -1.

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Clip Numbers

206 - 210, 251

Tip

Parallel lines have the same gradient.

Formula

$$\text{Gradient} = \frac{\text{difference in } y\text{'s}}{\text{difference in } x\text{'s}}$$

Questions

1) What are the gradient and y-intercept of:

a) $y = 4x - 3$

b) $y = 4 + 6x$

c) $y = -5x - 3$

2) Draw the graph of $y = 3x - 2$ for x values from -3 to 3 using a table.

c) $m = -5, c = -3$

b) $m = 6, c = 4$

a) $m = 4, c = -3$

ANSWERS: 1) a) $m = 4, c = -3$ b) $m = 6, c = 4$ c) $m = -5, c = -3$

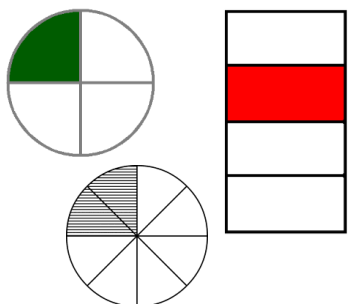
Half Term 3

UNDERSTANDING FRACTIONS

Key Concept

$$\frac{1}{4}$$

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16}$$



$$= 0.25$$

Key Words

Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).

Equivalence: Two fractions are equivalent if one is a multiple of the other.

Simplify: Cancel a fraction down to give the smallest numbers possible.

Examples

Simplify $\frac{3}{24}$

$$\frac{3}{24} \xrightarrow{\div 3} \frac{1}{8}$$

Write 0.34 as a fraction

1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0	•	3	4	

$$= \frac{34}{100} = \frac{17}{50}$$

Write $\frac{6}{7}$ as a decimal

$$\frac{6}{7} \rightarrow 7 \overline{) 6.000000}$$

0 . 8 5 7 1 4

6 4 5 1 3

What is $\frac{3}{5}$ of 45

$$45 \div 5 \times 3 = 27$$

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Clip Numbers

58, 59, 73, 74, 77

Tip

- A larger denominator does not mean a larger fraction.
- To find equivalent fractions multiply/divide the numerator and denominator by the same number.

Questions

- Simplify a) $\frac{42}{96}$ b) $\frac{64}{120}$
- Write as a decimal a) $\frac{2}{7}$ b) $\frac{3}{8}$
- Write as a fraction a) 0.48 b) 0.166 c) 0.308
- a) $\frac{3}{5}$ of 35 b) $\frac{2}{9}$ of 45 c) $\frac{5}{11}$ of 121

ANSWERS : 1) a) $\frac{16}{8}$ b) $\frac{16}{8}$ c) $\frac{15}{8}$ 2) a) 0.2857... b) 0.375 3) a) $\frac{25}{83}$ b) $\frac{500}{77}$ c) $\frac{250}{77}$ 4) a) 21 b) 10 c) 55

FRACTIONS

Key Concepts

Equivalent fractions have the same value as one another.

Eg. $\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$

A number multiplied by it's **reciprocal** gives the answer of 1. Or the reciprocal of a number is 1 over the number.

Eg. $\frac{1}{8}$ is the reciprocal of 8.
 $\frac{2}{5}$ is the reciprocal of $\frac{5}{2}$

$$1\frac{2}{3} + 2\frac{1}{4}$$

$$= \frac{5}{3} + \frac{9}{4}$$

$$= \frac{20}{12} + \frac{27}{12}$$

$$= \frac{47}{12}$$

$$= 3\frac{11}{12}$$

Convert into an improper fraction

Find a common denominator

Convert back into a mixed number

$$2\frac{2}{3} - 1\frac{1}{4}$$

$$= \frac{8}{3} - \frac{5}{4}$$

$$= \frac{32}{12} - \frac{15}{12}$$

$$= \frac{17}{12}$$

$$= 1\frac{5}{12}$$

$$1\frac{1}{3} \times 2\frac{3}{4}$$

$$= \frac{4}{3} \times \frac{11}{4}$$

$$= \frac{44}{12}$$

$$= 3\frac{8}{12}$$

$$2\frac{1}{3} \div 1\frac{3}{5}$$

$$= \frac{7}{3} \div \frac{8}{5}$$

$$= \frac{7}{3} \times \frac{5}{8}$$

$$= \frac{35}{24}$$

$$= 1\frac{11}{24}$$

Examples

Find the reciprocal of the second fraction....

...and multiply

Key Words

Fraction
 Equivalent
 Reciprocal
 Numerator
 Denominator
 Improper/Top heavy
 Mixed number

Calculate:

1) $1\frac{2}{3} + 2\frac{3}{4}$

2) $3\frac{3}{4} - 1\frac{1}{3}$

3) $3\frac{1}{5} \times 1\frac{2}{3}$

4) $1\frac{3}{5} \div 2\frac{7}{10}$

What is the reciprocal of:

5) $\frac{2}{3}$

7) 0.75

6) 9

FOUR OPERATIONS WITH FRACTIONS

Key Concept

Mixed numbers

These are made up of a whole number and a fraction.





$$4\frac{3}{5}$$

$$= \frac{4 \times 5 + 3}{5}$$

$$= \frac{23}{5}$$

Key Words

Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).

 <p>Add Sum Total All together Plus In all</p>	 <p>Multiply Product Times Twice Total Multiplied by</p>
 <p>Subtract Remain Difference Less than Fewer How many more Minus</p>	 <p>Divide Quotient Goes into Split Equally Each</p>

Tip

- A larger denominator **does not** mean a larger fraction.
- To find equivalent fractions multiply/divide the numerator and denominator by the same number.

Examples



$$\frac{3}{5} + \frac{2}{7}$$

Make the denominators the same

$$\begin{array}{c} \frac{3}{5} + \frac{2}{7} \\ \times 7 \quad \times 5 \\ \hline \frac{21}{35} + \frac{10}{35} = \frac{31}{35} \end{array}$$



$$\frac{3}{5} - \frac{2}{7}$$

$$\begin{array}{c} \frac{3}{5} - \frac{2}{7} \\ \times 7 \quad \times 5 \\ \hline \frac{21}{35} - \frac{10}{35} = \frac{11}{35} \end{array}$$



$$\frac{3}{5} \times \frac{2}{7}$$

Just multiply the tops and bottoms

$$= \frac{3 \times 2}{5 \times 7} = \frac{6}{35}$$



$$\frac{3}{5} \div \frac{2}{7}$$

Flip the second fraction and change to a times

$$\frac{3}{5} \times \frac{7}{2} = \frac{21}{10}$$

4 Rules
Fractions



Clip Numbers

61 – 70

Questions

1) $\frac{3}{5} + \frac{4}{15}$ 2) $\frac{2}{7} + \frac{5}{8}$ 3) $\frac{7}{9} - \frac{2}{5}$ 4) $\frac{3}{7} \times \frac{4}{9}$ 5) $\frac{3}{11} \div \frac{14}{22}$

ANSWERS: 1) $\frac{13}{15}$ 2) $\frac{56}{51}$ 3) $\frac{17}{45}$ 4) $\frac{21}{4}$ 5) $\frac{7}{3}$

FRACTIONS OF AN AMOUNT

Key Concepts

$$\frac{x}{y} \rightarrow \begin{array}{l} \text{Numerator} \\ \text{Denominator} \end{array}$$

Equivalent fractions have the same value as one another.

Eg. $\frac{1}{4} = \frac{2}{8} = \frac{3}{12}$



61, 63-70

Calculate $\frac{4}{5}$ of 65:

$$65 \div 5 = 13$$

$$13 \times 4 = 52$$

Divide by the denominator

Multiply this by the numerator

$\frac{4}{5}$ of a number is 52, what is the original number?

$$52 \div 4 = 13$$

$$13 \times 5 = 65$$

Divide by the numerator

Multiply this by the denominator

Examples

Order these fractions in ascending order:

$\frac{2}{5}$	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{7}{15}$
$\downarrow \times 6$	$\downarrow \times 15$	$\downarrow \times 5$	$\downarrow \times 2$
$\frac{12}{30}$	$\frac{15}{30}$	$\frac{25}{30}$	$\frac{14}{30}$
(1)	(3)	(4)	(2)

To be able to compare fractions we must have a **common denominator**

Key Words

Fraction
Equivalent
Reciprocal
Numerator
Denominator

1) Calculate $\frac{2}{7}$ of 56.

2) $\frac{3}{8}$ of a number is 36, what is the original number?

3) Order the following in ascending order: $\frac{2}{3}$ $\frac{5}{6}$ $\frac{3}{8}$ $\frac{7}{12}$

PERCENTAGES

Key Concepts

Calculating percentages of an amount without a calculator:

10% = divide the value by 10

1% = divide the value by 100

Calculating percentages of an amount with a calculator:

Amount \times percentage
as a decimal

**Calculating percentage
increase/decrease:**

Amount \times (1 \pm percentage
as a decimal)

Calculating a percentage – non calculator:

Calculate 32% of 500g:

$$10\% \rightarrow 500 \div 10 = 50$$

$$30\% \rightarrow 50 \times 3 = 150$$

$$1\% \rightarrow 500 \div 100 = 5$$

$$2\% \rightarrow 5 \times 2 = 10$$

$$\begin{aligned} 32\% &= 150 + 10 \\ &= 160\text{g} \end{aligned}$$

Calculating a percentage – calculator:

Calculate 32% of 500g:

$$\begin{aligned} \text{Value} &\times (\text{percentage} \div 100) \\ &= 500 \times 0.32 \\ &= 160\text{g} \end{aligned}$$

Percentage change:

Examples

A dress is reduced in price by 35% from £80. What is its **new price**?

$$\begin{aligned} \text{Value} &\times (1 - \text{percentage as a decimal}) \\ &= 80 \times (1 - 0.35) \\ &= £52 \end{aligned}$$

A house price appreciates by 8% in a year. It originally costs £120,000, what is the **new value** of the house?

$$\begin{aligned} \text{Value} &\times (1 + \text{percentage as a decimal}) \\ &= 120,000 \times (1 + 0.08) \\ &= £129,600 \end{aligned}$$



84-90

Key Words

Percent
Increase/decrease
Appreciate
Depreciate
Multiplier
Divide

- 1) Write the following as a decimal multiplier: a) 45% b) 3% c) 2.7%
- 2) Calculate 43% of 600 without using a calculator
- 3) Calculate 72% of 450 using a calculator
- 4a) Decrease £500 by 6%
- b) Increase 65g by 24%
- c) Increase 70m by 8.5%

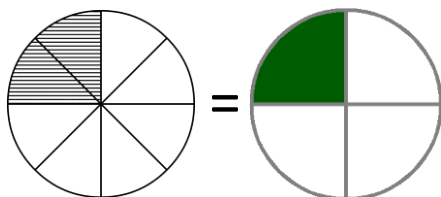
RATIO

Key Concept

2 parts \rightarrow $2:6$ \leftarrow 6 parts

$$=$$

$$1:3$$



$$= \frac{1}{3}$$

Key Words

Ratio: Relationship between two numbers.

Part: This is the numeric value '1' of, would be equivalent to.

Simplify: Divide both parts of a ratio by the same number.

Equivalent: Equal in value.

Convert: Change from one form to another.

Examples

Simplify $60 : 40 : 100$

This could have been done in one step by dividing by 20.

$$\div 10$$

$$6 : 4 : 10$$

$$\div 2$$

$$3 : 2 : 5$$

Write $2 : 5$ in the form $1 : n$

$$\begin{array}{ccc} & 2 : 5 & \\ \div 2 \swarrow & & \searrow \div 2 \\ & 1 : 2.5 & \end{array}$$

Share £45 in the ratio $2 : 7$

$$45 \div 9 = 5$$

$$\text{£}10 : \text{£}35$$

$$2 : 7$$

5	5
5	5
=10	5
	5
	5
	5
	5
	=35

Joy and Martin share money in the ratio $2 : 5$. Martin gets £18 more than Joy. How much do they each get?

$$\text{£}12 : \text{£}30$$

$$\begin{array}{cc} 2 : 5 \\ \begin{array}{|c|c|} \hline 6 & 6 \\ \hline 6 & 6 \\ \hline 6 & 6 \\ \hline 6 & 6 \\ \hline \end{array} \\ 18 \div 3 = 6 \\ \text{---} \\ =12 \quad =30 \end{array}$$



Clip Numbers

328 – 335

Tip

Its often useful to write the letters above the ratio. This helps you keep the order the correct way round.

Questions

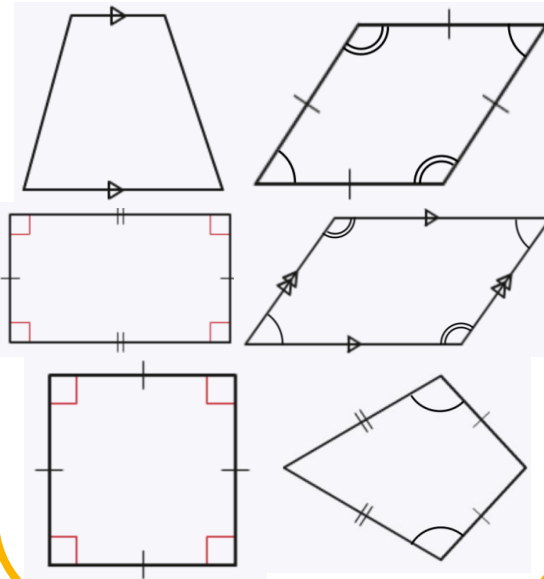
- 1) Simplify a) $45 : 63$ b) $66 : 44$ c) $320 : 440$
- 2) Write in the form $1 : n$ a) $5 : 10$ b) $4 : 6$ c) $x : x^2 + x$
- 3) Share 64 in the ratio $3 : 5$ 4) Write the ratio $1 : 4$ as a fraction.

ANSWERS: 1) a) 5 : 7 b) 3 : 2 c) 8 : 11 2) a) 1 : 2 b) 1 : 1.5 c) $1 : x + 1$ 3) 24 : 40 4) $\frac{1}{5}$

Half Term 4

PROPERTIES OF SHAPES

Key Concept Quadrilaterals



Key Words

Angle: This is formed by two lines, joined by a common endpoint.

Symmetry: A shape has symmetry if there is a line which forms two equal parts which are a mirror image of each other.

Reflection: This is where a shape is flipped.

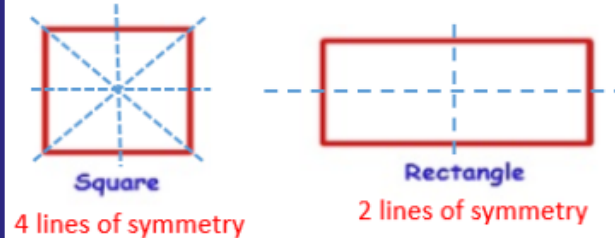
Rotation: This is where a shape is turned.

Tip

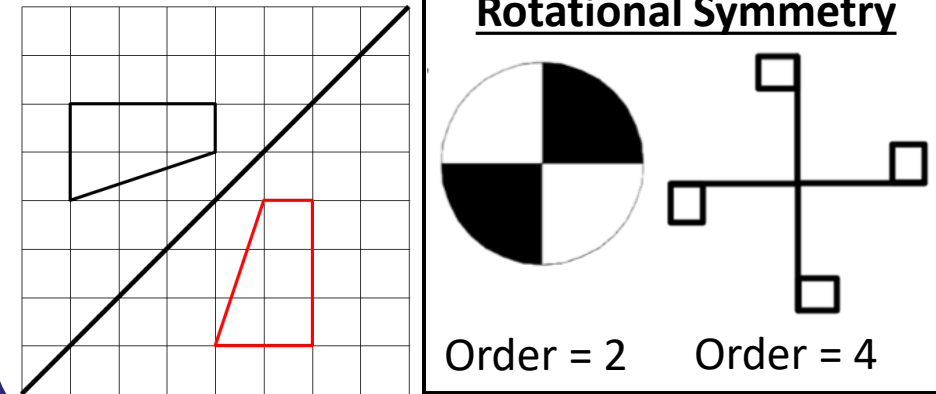
- The smallest the order of rotational symmetry can be, is 1.
- To see if a line of symmetry works fold along the line and see if the both halves lie exactly on top of each other.

Examples

Lines of symmetry and reflection



Rotational Symmetry



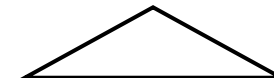
hegartymaths
Clip Numbers
457-460, 639-649,
822-828

Questions - For the shapes below draw on their lines of symmetry and state their order of rotational symmetry.

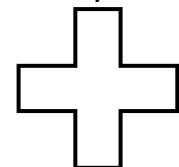
1)



2)



3)



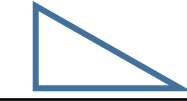




ANSWERS: 1) 2 lines of symmetry, order = 2 2) 1 line of symmetry, order = 1 3) 4 lines of symmetry, order = 4.

PERIMETER

Key Concept

2D Shapes

	Parallelogram
	Trapezium
	Right-angled triangle
	Isosceles triangle
	Equilateral triangle

Key Words

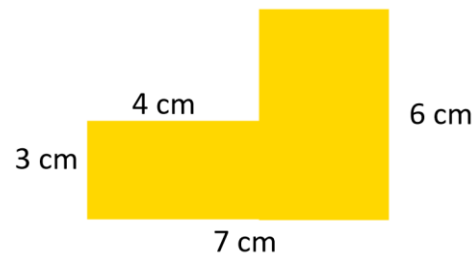
Perimeter: The distance around the outside of the shape.
Unit of measure: This could be any unit of length cm, inch, m, foot, etc.

Dimensions: The lengths which give the size of the shape.

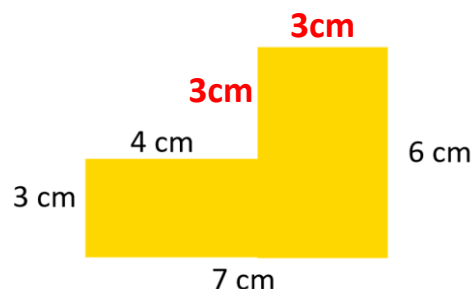
Compound shape: A shape made up of two composite shapes

Example

Find the perimeter



Step 1 – Find the missing lengths.



Step 2 – Add the lengths

$$3 + 4 + 3 + 3 + 6 + 7 = \underline{26 \text{ cm}}$$

 hegartymaths

Clip Numbers

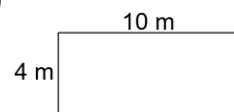
534-550, 691, 822

Tip

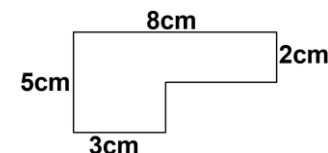
- Always include units with your answer.

Questions – Find the perimeter of each shape to 1dp

1) a)



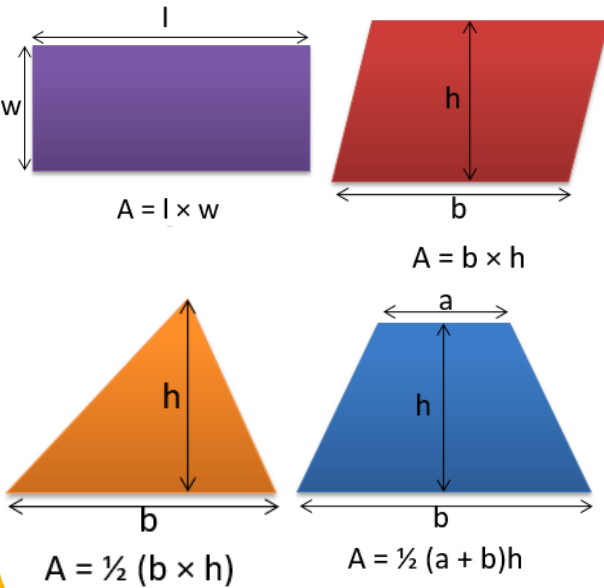
b)



ANSWERS: 1) a) 28 m b) 26 cm

AREA

Key Concepts Area



Key Words

Area: The amount of square units that fit inside the shape.

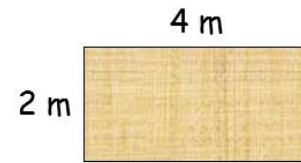
Perimeter: The distance around the outside of the shape.

Dimensions: The lengths which give the size of the shape.

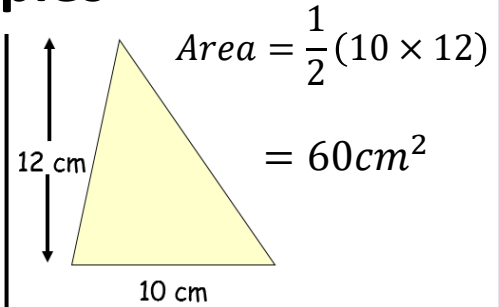
Shapes:

Rectangle, Triangle, Parallelogram, Trapezium, Kite.

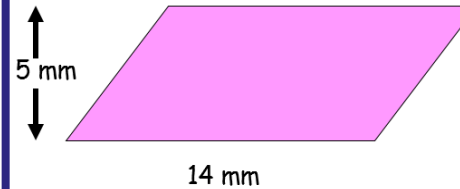
Examples



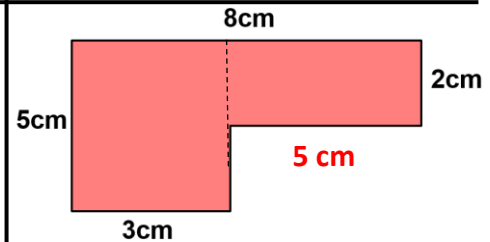
$$Area = 2 \times 4 = 8m^2$$



$$Area = \frac{1}{2} (10 \times 12) = 60cm^2$$



$$Area = 5 \times 14 = 70mm^2$$



$$Area = (5 \times 3) + (2 \times 5) = 25cm^2$$

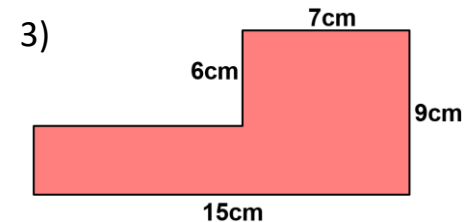
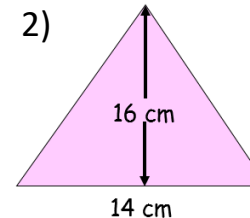
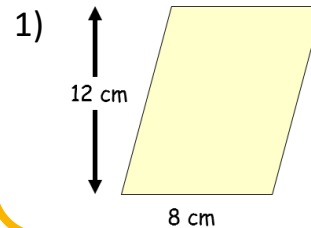
hegartymaths

Clip Numbers
554 – 559

Tip

Always remember units. These units are squared for area. mm^2 , cm^2 , m^2 , etc

Questions – Find the area.

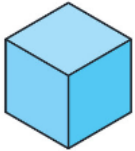


ANSWERS: 1) $96cm^2$ 2) $112cm^2$ 3) $87cm^2$

3D SHAPES, CAPACITY AND VOLUME

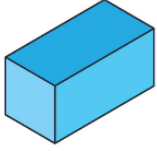
Key Concept

Cube



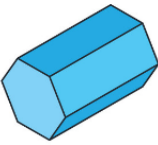
Faces – 6
Edges – 12
Vertices – 8

Cuboid



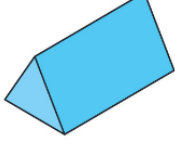
Faces – 6
Edges – 12
Vertices – 8

Hexagonal Prism



Faces – 8
Edges – 18
Vertices – 12

Triangular Prism



Faces – 5
Edges – 9
Vertices – 6

Key Words

Volume: The amount of space that an object occupies.

Capacity: The amount of space that a liquid occupies.

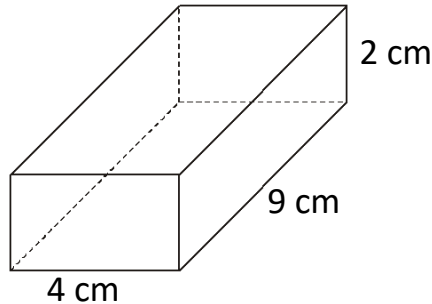
Cuboid: 3D shape with 6 square/rectangular faces.

Vertices: Angular points of shapes.

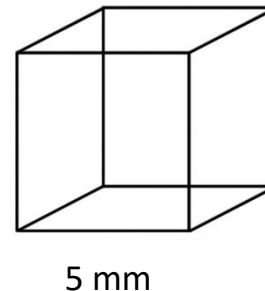
Face: A surface of a 3D shape.

Edge: A line which connects two faces on a 3D shape.

Examples



$$\begin{aligned} \text{Volume} &= 4 \times 9 \times 2 \\ &= 72\text{cm}^3 \end{aligned}$$



$$\begin{aligned} \text{Volume} &= 5 \times 5 \times 5 \\ &= 125\text{mm}^3 \\ \text{or} \\ \text{Volume} &= 5^3 \\ &= 125\text{mm}^3 \end{aligned}$$

 hegarty**maths**

Clip Numbers

568-571,698,699

Tip

Remember the units are cubed for volume.

Formula

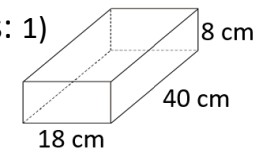
Cuboid Volume = $l \times w \times h$

Cube Volume = $l \times w \times h$ or

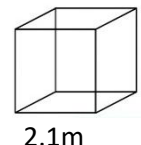
Cube Volume = w^3

Questions

Find the volume of these shapes: 1)



2)



ANSWERS: 1) 5760 cm³ 2) 9.261 m³

PLANS AND ELEVATIONS

Key Concepts

A 3 dimensional shape can be mathematically drawn from **three view points**:

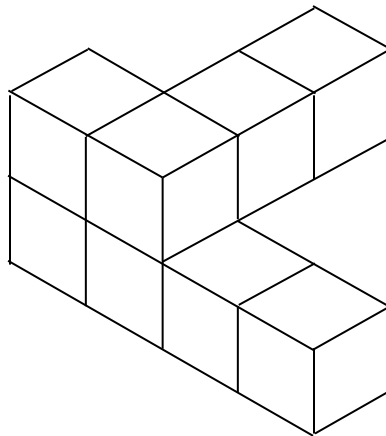
Side view

Front view

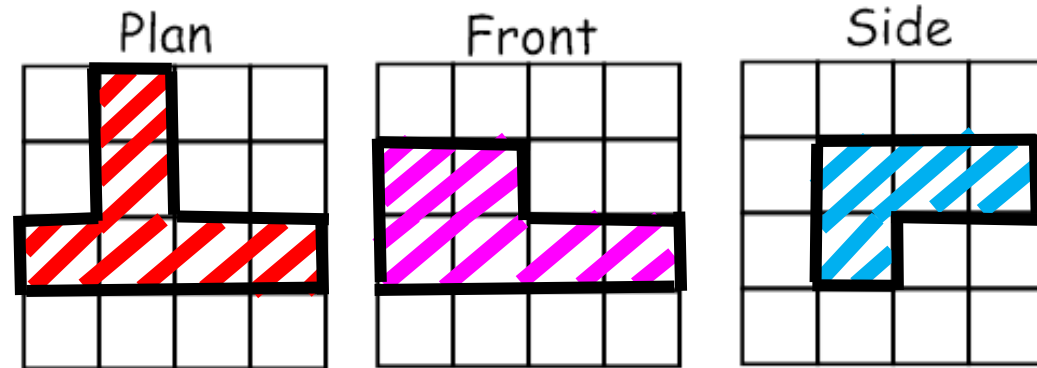
Plan view – from above

They are drawn as 2 dimensional representations

Draw this 3D shape from the side view, the front view and the plan view.



Examples



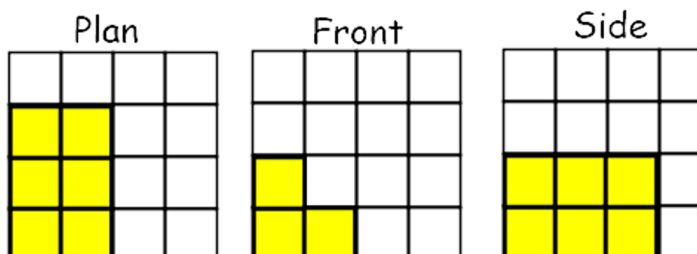
Key Words

Elevation

Plan

Side

Front



Sketch the 3D shape that has these three views.

MEASURING AND DRAWING ANGLES

Key Concepts

Types of angle

There are four types which need to be identified:

- Acute
- Obtuse
- Reflex
- Right - angled

Measuring – Align centre, Align 0, count up

Drawing - Measure, mark, connect, label

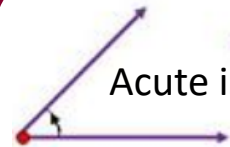
 hegartymaths

455, 456

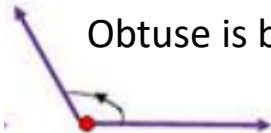
Key Words

Acute
Obtuse
Right angle
Reflex

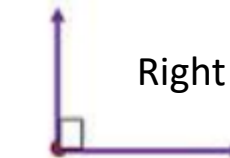
Examples



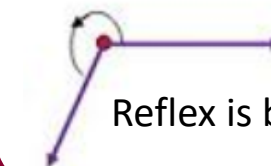
Acute is less than 90°



Obtuse is between 90° and 180°

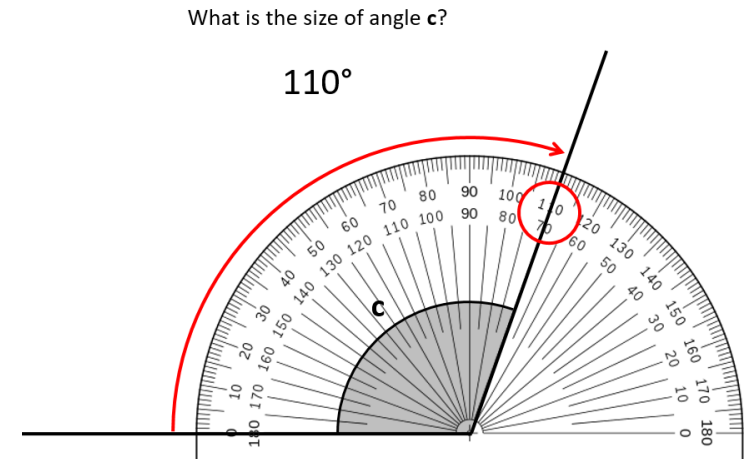


Right angled is 90°



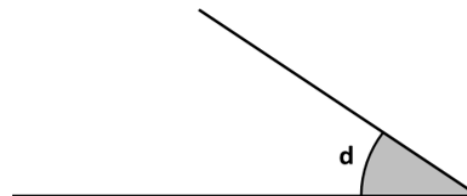
Reflex is between 180° and 360°

Construct an angle of 150° :

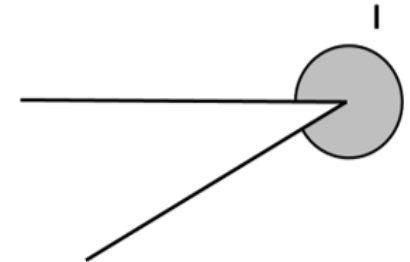


Questions

1)



2)



Half Term 5

ANGLE FACTS

Key Concepts

Angles in a **triangle equal 180°**.

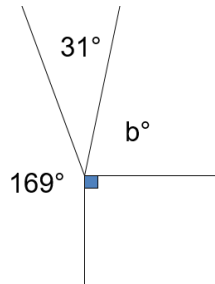
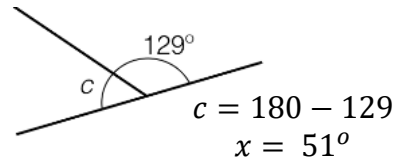
Angles in a **quadrilateral equal 360°**.

Vertically opposite angles are equal in size.

Angles on a **straight line equal 180°**.

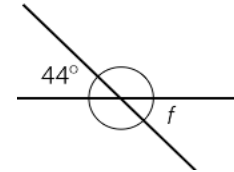
Base angles in an isosceles triangle are equal.

Examples

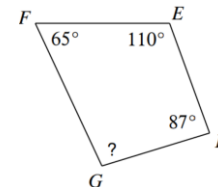


$$169^\circ + 31^\circ + 90^\circ = 290^\circ$$

$$360^\circ - 290^\circ = 70^\circ$$

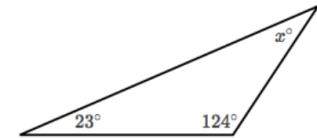


$$f = 44^\circ$$



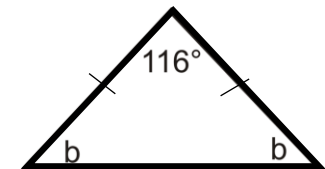
$$? = 360 - (65 + 110 + 87)$$

$$? = 98^\circ$$



$$x = 180 - (23 + 124)$$

$$x = 33^\circ$$



$$b = (180 - 116) \div 2$$

$$b = 32^\circ$$

Key Words

Angle

Vertically opposite

Straight line

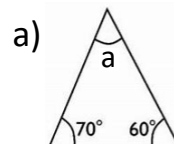
Isosceles triangle

Quadrilateral

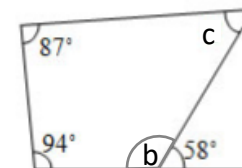
Interior angles

Questions

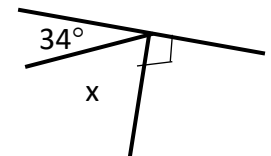
Calculate the missing angle:



b)



c)



TRIANGLE CONSTRUCTIONS

Key Concepts

Construction – drawing of actual scale

ASA – Angle, Side, Angle

SAS – Side, Angle, Side

SSS – Side, Side, Side

Key Words

Angle
Construct
Compass
Arc
Angles
Protractor

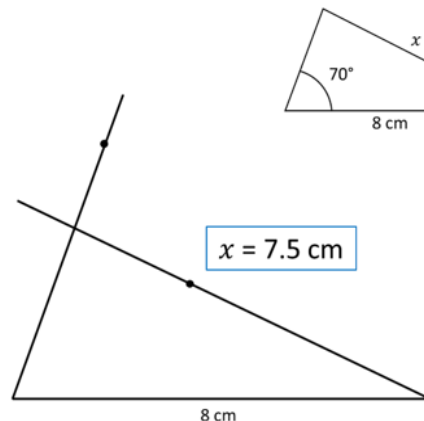


477-480, 481-483

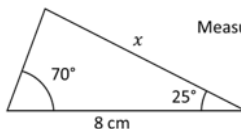
Examples

Constructing Triangles: Angle Side Angle

①



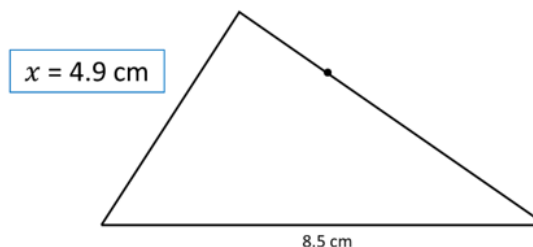
Construct a copy of this triangle.



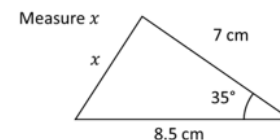
Measure x

Constructing Triangles: Side Angle Side

②



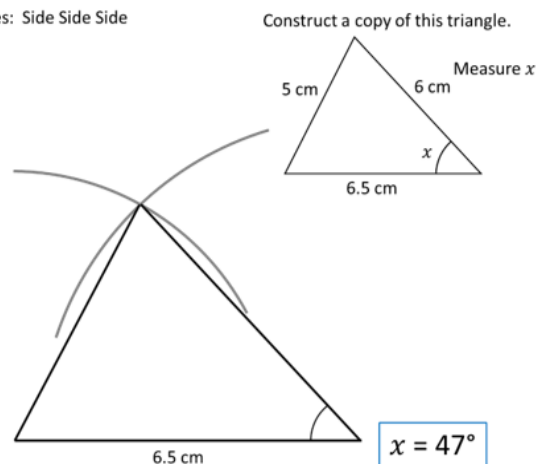
Construct a copy of this triangle.



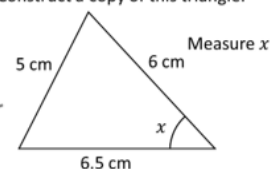
Leave construction lines.

Constructing Triangles: Side Side Side

③



Construct a copy of this triangle.

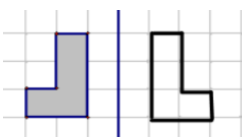


Leave construction lines.

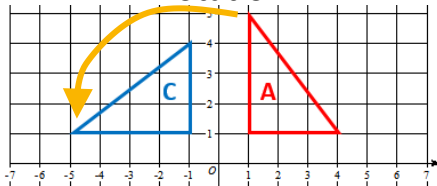
CO-ORDINATES AND TRANSFORMATIONS

Key Concept

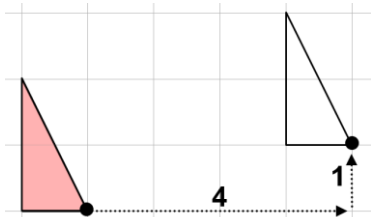
Reflection



Rotation



Translation



Key Words

Co-ordinate: A pair of numbers which describe the position on a grid.

Transformation: This means the shape has 'changed'.

Reflection: This means a shape has been flipped.

Rotation: This means a shape has been turned.

Translation: This means a *movement* of the shape.

Tip

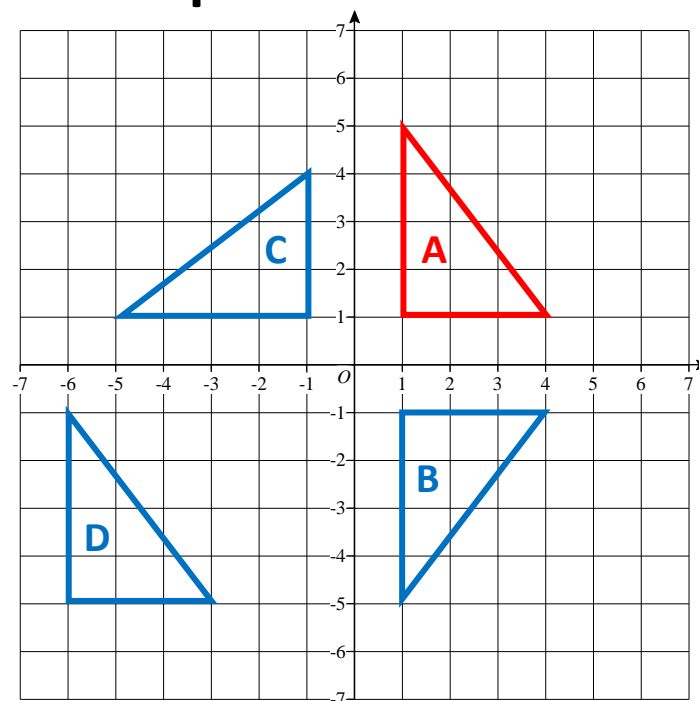
- Use **tracing paper** to avoid mistakes.
- When describing transformations, look at how many marks are available and see if you have put enough to get the marks.

Examples

a) Reflect A in the x-axis, label it B.

b) Rotate A 90°, anti-clockwise about (0,0), label it C.

c) Translate A in the vector $\begin{pmatrix} -7 \\ -6 \end{pmatrix}$, label it D.



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Clip Numbers

199, 205, 637-657

Questions

Draw a grid like the one above.

Plot a triangle with vertices (6,2), (3, 2) and (4, 5).

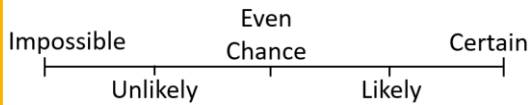
a) Reflect the triangle in the y-axis. b) Translate the triangle $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

ANSWERS: a) (-6,2), (-3,2) and (-4,5) b) (1,1), (0,-2) and (3,-2)

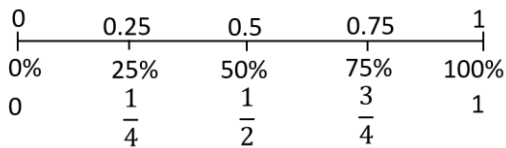
INTRODUCING PROBABILITY

Key Concept

Chance



Probability



Probabilities can be written as:

- Fractions
- Decimals
- Percentages



Clip Numbers

349 - 359

Key Words

Probability: The chance of something happening as a numerical value.

Impossible: The outcome cannot happen.

Certain: The outcome will definitely happen.

Even chance: There are two different outcomes each with the same chance of happening.

Expectation: The amount of times you expect an outcome to happen based on probability.

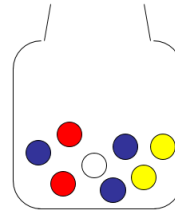
Tip

Probabilities always add up to 1.

Formula

Expectation
= Probability \times no. of trials

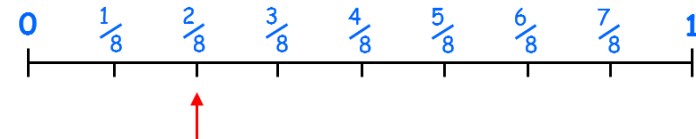
Examples



- 1) What is the probability that a bead chosen will be **yellow**.
Show the answer on a number line.

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(\text{Yellow}) = \frac{2}{8} = \frac{1}{4}$$



- 2) How many **yellow** beads would you **expect** if you pulled a bead out and replaced it 40 times?

$$\frac{1}{4} \times 40 = \frac{1}{4} \text{ of } 40 = 10$$

Questions

In a bag of skittles there are 12 red, 9 yellow, 6 blue and 3 purple left.
Find: a) P(Red) b) P(Yellow) c) P(Red or purple) d) P(Green)

ANSWERS: 1) a) $\frac{12}{30} = \frac{2}{5}$ b) $\frac{9}{30} = \frac{3}{10}$ c) $\frac{21}{30} = \frac{7}{10}$ d) 0

Half Term 6

TYPES OF DATA AND GRAPHS

Key Concepts

Qualitative data: data collected that is described in words **not** numbers.
e.g. race, hair colour, ethnicity.

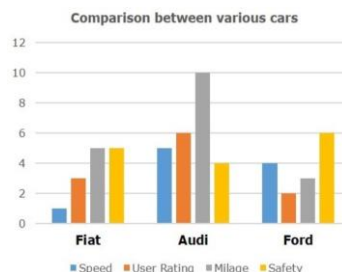
Quantitative data: this is the collection of numerical data that is either discrete or continuous.

Discrete data: numerical data that is categorised into a finite number of classifications.

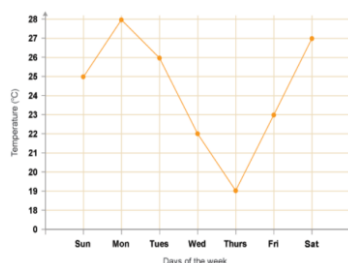
e.g. number of siblings in a family, shoe size, .

Continuous data: numerical data that can take any value. This data is usually measured on a large number scale.
e.g. height, weight, time, capacity.

Comparative bar charts



Line graphs



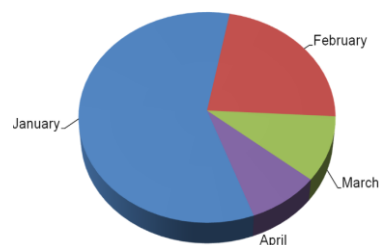
Examples

Tally charts

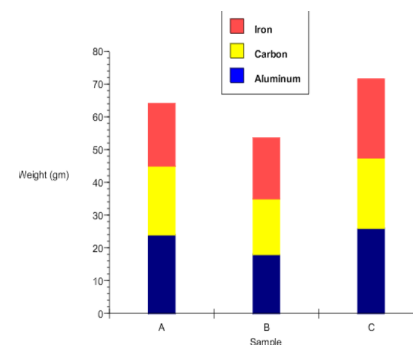
Colour	Tally	Frequency
Red		13
Blue		9
White		24
Black		12
Other		9

Pie charts

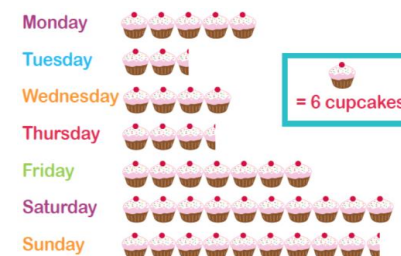
Sales split month wise



Composite bar charts



Pictograms



hegarty^{maths}

425,426,427,
430-433,442

Key Words

Data
Discrete
Continuous
Qualitative
Quantitative
Graph

What types of data is each of the following?

- 1) Eye colour
- 2) Time it takes to run 100m
- 3) Number of goals scored in a match
- 4) Length of a car (to the nearest cm)
- 5) Number of pets a person owns

ANSWERS: 1) Qualitative
2) Continuous, quantitative
3) Discrete, quantitative
4) Continuous, quantitative
5) Discrete, quantitative

BAR CHARTS AND PICTOGRAMS

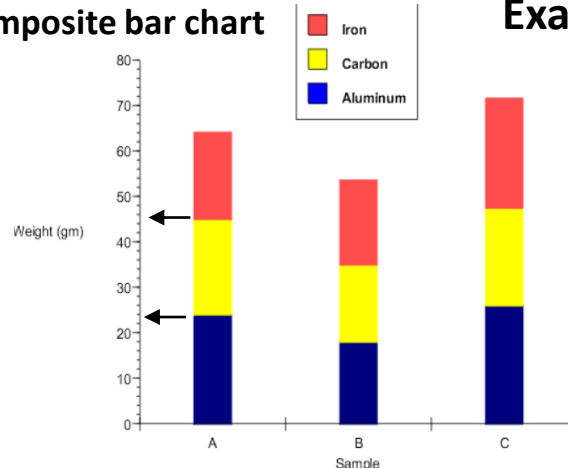
Key Concepts

Bar charts are a visual representation of **categorical data**.

Composite bar charts are bar charts that display multiple data points stacked on top of one another.

Pictograms use an image relating to a physical object to represent an amount. A **key** must be included to show the value of each picture.

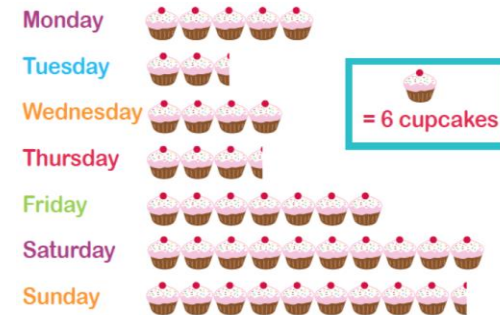
Composite bar chart



- How much aluminium is in sample A? **24g**
 - How much carbon is in sample A? **22g**
- $46 - 24 = 22g$
 Highest value for carbon in sample A. Lowest value for carbon in sample A.

Examples

Pictogram



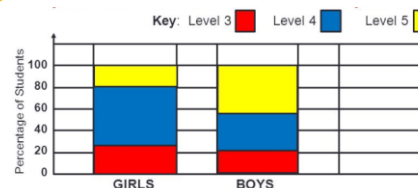
- How many cupcakes were sold on Monday?
 $5 \times 6 = 30$ cupcakes
- What does half a cupcake represent on the pictogram?
 $6 \div 2 = 3$ cupcakes
- How many cupcakes were sold on Thursday?
 $3.5 \times 6 = 21$ cupcakes



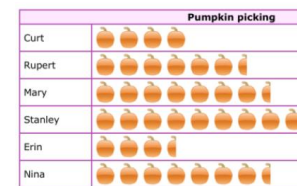
425-426

Key Words

Bar chart
Composite
Pictogram
Key
Categorical
Data set



- What percentage of boys are level 3?
- What percentage of girls are level 4?



Each = 2 pumpkins

- How many pumpkins were picked by Stanley?
- What does half a pumpkin represent?
- How many pumpkins were picked by Erin?

AVERAGES

Key Concepts

There are three types of **average** that we use to analyse and compare data. We can calculate averages from a **discrete** data set.

Mode The most common value that appears in the list.

Median Once ordered, the middle value.

Mean
$$\frac{\text{Total of all data}}{\text{Number of pieces of data}}$$

The **range** is used to analyse the **spread** of a data set or how **consistent** the data is.

Range
$$\text{largest data value} - \text{smallest data value}$$



Clip Numbers
400 – 429

Key Words

Frequency: Total.

Mean: Total of data divided by the number of pieces of data.

Mode: The value that occurs most frequently.

Median: Middle number when they are in order.

Range: Difference between the largest and smallest values.

Examples

5, 9, 9, 9, **11**, 12, 13, 15, 16

Averages

$$\text{Mean} = \frac{5 + 9 + 9 + 9 + 11 + 12 + 13 + 15 + 16}{9} = \frac{99}{9} = 11$$

Median = 11 (The middle number shown above)

Mode = 9 (This number occurs most often)

Measure of Spread

$$\text{Range} = 16 - 5 = 11$$

(A bigger range means the data is more spread out)

Questions

1) Find the mean, mode, median and range of:

a) 3, 12, 4, 6, 8, 5, 4 b) 12, 1, 10, 1, 9, 3, 4, 9, 7, 9

2) For the table:

- a) Draw a bar chart to show the data.
b) Work out the mean of the data.

Age	Frequency
11	17
12	11
13	8

ANSWERS: 1) a) Mean = 6, Mode = 4, Median = 5, Range = 9 b) Mean = 6.5, Mode = 9, Median = 8, Range = 11 b) 11.75